AN ANALYTIC APPROACH TO DESIGN CENTRIFUGAL IMPELLER GEOMETRY

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ABSTRACT

Two different types of analytic functions have been evolved to define the centrifugal impeller geometry. A parameter called wrap angle was used in these analytic functions to vary the impeller blade shape. The relative advantages of one analytic function over the other is highlighted by comparing the blade geometry, channel area variation and derivatives of important geometric parameters affecting the flow. The impellers generated from the evolved analytic equations were theoretically analyed for hub to shroud and blade to blade flows. The performance of the impellers were compared from the aspects of boundary layer parameter and blade loading. First and second analytic functions for large wrap angles and only first analytic function for small wrap angles were found suitable to define the blade geometry. For large axial lengths impeller having small wrap angle was found advantageous. Wrap angles of the order slightly greater than the fluid deflection were found advantageous from the aspect of derivatives of important geometric parameters.

NOMENCLATURE

A,B,C,D	constants used in the analytic equation
A' ,B'	constants used in defining the hub contour
a	area
B_1	blade loading coefficient
b	impeller width
C_f	skin friction coefficient
L	axial length of the impeller
m	meridional distance
r	radius
r_{cs}	radius of curvature of shroud
S	surface distance
(r,θ,z)	cylindrical co-ordinate system
W	relative flow velocity
(ζ,η)	transformed co-ordinate system
α_0	parameter for shroud contour
β	blade angle w.r.t. axial direction
δ	wrap angle

SUBSCRIPTS

b	blade
c	curvature
h,Hu	hub
o	offset
s,Sh	shroud
t	total
1	impeller inlet
2	impeller outlet
-	normalized value

1. INTRODUCTION

The efficiency of centrifugal compressor stage consisting of an impeller, diffuser and a volute casing depends partly on the impeller design. A good uniform flow at impeller outlet increases the compressor stage efficiency due to reduction in mixing losses. The impeller performance characteristics depend on meridional channel geometry and blade geometry. Both these geometrical quantities are inter dependent and influence each other.

A potential flow analysis for a given impeller geometry including blade shape would give the basic velocity distribution. This velocity distribution can be used to evaluate loading distribution and viscous effects arising out of velocity gradients. Boundary layer characteristics for the velocity distribution would show the supremacy of one geometry over the other.

The design of the impeller is essentially to optimise performance with regard to above aspects of flow characteristics. In this, the use of an analytic function to generate the blade geometry would greatly help in defining the passage and evaluating flow properties through the same. The type of analytic function used, should fit in with the geometrical boundary conditions imposed at inlet and outlet for an overall design requirement of mass flow rate and pressure ratio. And in addition, it should provide for a variation of geometrical parameters of the passage, for an optimal characteristic and efficient performance. Which can be evaluated through an analysis. Such analytic functions and criteria for design parameters are evolved and their relative advantages are highlighted in this paper.

Use of an analytic function to generate the blade geometry would greatly help in optimising the impeller performance with reference to boundary layer characteristics and blade loading. Use of analytic methods to help in designing geometric shapes have been attempted earlier. Came (1) described the impeller blade shape by means of analytic surfaces. The method involves a large number of patches albeit through simpler equations, but it does not establish a way to control the flow for a desired characteristic. Whitfield et al. (2) following a practice at NGTE have used equations of ellipse to define camber line shape, as well as hub and shroud contours. A lean of the blade, was also introduced, which was to increase linearly at impeller outlet from hub to shroud. Apart from simplicity claimed over the surface patches method, there does not seem to be any other advantage in obtaining a better performance of the flow through the impeller. Krain (3) also used an analytic equation for generating blade geometry. A comparison of two different duty requirements of design have been compared without indicating the exact nature of equation used. Use of B-splines have become common to define free form surfaces like centrifugal impeller blade geometries. The surface is generated with number of spline curves. Each curve is expressed by a polynomial with varying degree. These curves are defined by a set of points called control points (4,5,6). The continuty constraints like position, first derivative and second derivatives are satisfied at the junction points. Bezier curves are special case of B-splines.

The choice of an analytic function should eliminate large blade curvatures, and obtain an even or appropriate distribution of blade loading or diffusion. These are in turn dependent on passage area variation, first and second derivatives of geometric co-ordinates and their variation across or through the passage. This paper describes the use of analytic functions evolved to generate blade shapes in centrifugal impellers, which are optimised from aspects of blade geometry, channel area variation and derivatives of important geometric parameters affecting the flow. The impellers generated from the evolved analytic equations were theoretically analysed for performance. Blade to blade and hub to shroud flow analysis were carried out. The impeller performance were compared from the aspects of boundary layer parameter and blade loading.

2. **AERODYNAMIC DESIGN**

Design methodology for impellers works on iterative procedure with following aspects, viz.

Design of impeller inlet geometry for a given specific speed and hub/shroud diameter ratio to limit relative inlet tip Mach number.

Surface velocity distribution giving rise to loading distribution and cross channel gradients.

Boundary layer characteristics of the surface velocity distribution.

Empirical modeling of real flow phenomenon such as secondary flows and viscous effects through slip, jet-wake pattern, etc. These are in turn dependent on cross channel and meridional gradients.

Loss evaluation due to surface friction, diffusion, separation, leakage and mixing process resulting in a uniform diffuser outlet condition.

The performance of an impeller is evaluated by characterising the flow in terms of surface velocity distribution and boundary layer parameters which are calculated from an analysis of flow through the impeller passage. These characteristics are in turn influenced by the channel area variation and blade curvature, which depend mainly on the first and second derivatives of the blade geometrical co-ordinates. An example of a given pressure ratio of 6.0 with a mass flow of 1 kg/sec. has been chosen to evaluate the efficacy of the evolved analytic functions in influencing the flow properties through smooth variations of geometric parameters. The inlet dimensions and speed are fixed from specific speed and tip Mach number considerations.

Flow entering from atmosphere into impeller inlet behaves like a forced vortex. Assuming constant meridional velocity from hub to shroud and a forced vortex flow at impeller inlet, it can be shown that

$$\tan\beta_1 / r = \text{constant} \qquad (1)$$

For compressors with integral inducers some results with aircraft superchrgers tests showed (7) that a positive incidence of about 10 degrees gives the best results. Tests with a range of freon compressors (8) showed maximum efficiency at a positive incidence of around 4 degrees. With the assumed value of incidence the variation of blade angle at impeller inlet can be evaluated. The blade angles at inlet hub ans shroud sections are used in the boundary conditions to evaluate the constants in the analytic equations.

A back sweep of 30 degrees is also assumed and the corresponding. outlet diametercommensurate with the chosen speed is determined to meet the requiredpressure ratio. From the overall geometry and speed the inlet and outlet velocity triangles are be derived. These stipulate the fluidangles at inlet and outlet fro aerodynamic considerations, and lead to the designation of blade inlet and outlet angles from the considerationof incidence and slip.

Losses within the blade channel give rise to non-uniform velocity distribution at impeller outlet. The magnitude of impeller loss may be decreased by suitably shaping the geometry. Decrease in shroud clearance will reduce the leakage loss. Surface frictional loss can be reduced by making the blade surface smooth and having shorter blade length. Greater losses would always occur near the shroud stream tube than in any other interior stream tubes. This reduces the meridional velocity at impeller exit near the shroud end. The meridional velocity profile at impeller exit can be improved by increasing the work input to the fluid in the region of shroud in order to compensate for the increased total pressure loss which occur in this region. This implies that impeller blade outlet angle at the shroud and at the hub should be different. With backsweep, the blade angle at impeller outlet shroud end with reference to radial line should be smaller than at the hub to get greater work input at the shroud. In order to get a smooth blade trailing edge, this variation of blade outlet angle from hub to shroud should be accompanied by a small lean of the blade at outlet. If we consider in terms of wrap angle, this would stipulate that shroud camber line should have a slightly smaller wrap angle as compared to hub camber line. The evolved analytic functions are capable of building this variation of blade shape at trailing edge without deterioration of passage shape inside.

3. MERIDIONAL SHAPE

The impeller performance characteristics depend on meridional channel geometry also in addition to blade geometry. Both these geometrical quantities are inter-dependent and influence each other.

The hub geometry of the impeller in the meridional plane is defined by an analytic equation of the form

$$\frac{(z_0 + z_h)^2}{a^2} + \frac{(r_2 + r_0 - r_h)^2}{b^2} = 1 \qquad (2)$$

Where A' and B' are constants evaluated from the slope of the hub contour at impeller inlet and outlet and total axial length of the impeller.

These constants are given by

$$A'^2 = \frac{(L + Z_0)^2 \ (r_0 + r_2 - r_{1h})^2 - Z_o^2 \, r_0^2}{(r_0 + r_2 - r_{1h})^2 - r_0^2}$$

and

$$B'^2 = \frac{r_o^2 \, Z_o^2 - (r_o + r_2 - r_{1h})^2 \, (L+Z_o)^2}{}$$

$$z^2_o - (L+Z_0)^2$$

Different hub geometries can be obtained by varying Z_o and r_o for a given overall geometry. This would also give different slopes of the hub profile at inlet and outlet in the meridional plane for the specified overall geometries like r_2 , r_{1h} , r_{1s} and L .

Similarly the shroud geometry in the meridional plane is defined by another analytic equation which satisfies the equation of a circle

$$r_{cs}^2 = Z_s^2 + (r_2^2 + r_{cs} \sin\alpha_o - r_s)^2$$

where $\tan(\alpha_o+\lambda/2) = (L-b_2)/(r2-r1s)$

$$\lambda = 2\sin^{-1} \frac{\sqrt{(L-b_2)^2 + (r_2-r_{1s})^2)}}{2 r_{cs}}$$

Different shroud geometry and slopes of the shroud at inlet and outlet are obtained by varying the radius of curvature, r_{cs} for a given overall meridional geometry like L, b_2, r_{1s} and r_2.

4. IMPELLER BLADE SHAPE

The vane design of the impeller should conform to the inlet and outlet velocity triangles as obtained above. The impeller blades are designed from consideration of surface velocity distribution calculated from theoretical methods such as streamline curvature. These methods analyse the flow in two dimensional axisymmetric plane or hub/shroud mean stream sheet plane. Hence the impeller geometry in cylindrical co-ordinate system (r,θ,z) is conveniently transformed into a rectilinear co-ordinate system (ζ,η) , in which the fluid dynamic conditions are preserved. The blade angles everywhere are preserved through this transformation (Fig. 1)

$$d\zeta = dm/r \text{ and } d\eta = d\theta \qquad (2)$$

It is now required to develop a camber line on which blade thickness (generally uniform from inlet to outlet) can be put to arrive at the blade shape. Different types of analytic functions were tried to define the camber line shape. Two analytic equations designated as Type-A and Type-B were chosen out of these and are dealt here. The analytic equation for blade camber line should meet the requirement of blade inlet angle and outlet angle obtained from velocity triangles. It should uniquely define the camber line shape from inlet to outlet and have smooth continuous variation of derivatives. The two types of equations described here are found to have these properties. Amongst these two, the superiority of one over the other is established.

Type-A analytic equation is given by

$$\eta = e^{A\zeta} + B\,\zeta^2 + C\,\zeta + D$$

Type-B analytic equation is given by

$$\eta = A\,e^{B\zeta} + C^{-D\zeta}$$

Where A, B, C and D are constants which are evaluated from the boundary conditions at inlet and outlet namely

at $\zeta= 0$, $\eta = 0$ and $d\eta/d\zeta = \tan \beta_{1b}$

at $\zeta = L$, $\eta = \delta$ and $d\eta\backslash d\zeta = \tan\beta_{2b}$

The parameter d is called the wrap angle. The wrap angle is defined as the included angle between the radial lines passing throughthe leading and trailing edges of the blades either at hub or at shroud.This is an important parameter used to control the rate of diffusion along the channel length.

5. **PASSAGE ANALYSIS**

For each analytic equation different impeller blade shapes were generated by varying the wrap angle from 10 deg. to 70 deg. Amongst these shapes, two impellers with wrap angle of 20 deg. and 60 deg. were considered for analysis. The impellers generated from Type-A analytic equation are designated by A-20 and A-60, whereas those from Type-B analytic equation are designated by B-20 and B-60. The numerals attached to the alphabets indicate the wrap angle in degrees. The isometric views of these impellers are shown in Fig. 2. From a cursory look the impellers A-60 and B-60 look similar, owing to large wrap angle. Impeller A-20 has large curvature and shorter blade length as compared to impeller A-60. Impeller B-20 looks different from impeller A-20. The two types of analytic equations used give different shapes of the impeller blades at low wrap angle.

The flow in centrifugal impeller enters axially and leaves radially. As the radius changes from inlet to outlet the camber line shape in the cylindrical coordinate system r,θ,Z as shown in Fig. 3 also changes. The camber line shape θ is plotted against the non-dimensional radius ratio. For both types of impeller derived from analytic equation of Type-A the camber line varies smoothly from inlet to outlet with respect to radius. For the impeller A-20 the local value of θ will reach a value greater than the given wrap angle within a short distance from the leading edge. Though such characteristic was not noticed with Type-B function, a sharp change in θ value very close to the leading edge was noticed for impeller B-20. Such a variation of camber line shape of the blade would not be suitable for good aerodynamic performance. Impellers derived from two types of analytic equations at higher wrap angles show very similar variation of shape with continuous increase in θ from inlet to outlet.

It is worth looking at the camber line shape of the blade in the transformed plane ζ-η from the point of flow analysis. As the flow analysis is generally carried out in this two-dimensional axisymmetric plane. The variation of local value of η against normalised value of ζ for the impellers derived from both types of analytic equations are shown in Fig. 4 . A monotonic variation of ζ for a given value of η was noticed for impellers A-60 and B-60. This fact is useful in defining the blade geometry for manufacture as well as for theoretical analysis. The blade geometry can be defined by straight line elements connecting hub and shroud points and having a constant θ along these lines. These straight line elements can be used as quasi orthogonals for the flow analysis in the meridional plane. One cannot obtain values of ζ for certain values of η for impeller A-20; therefore a constant θ quasi-orthogonals cannot be defined. As the difference in ζ values at hub and shroud are large for a given value of η, for impeller B-20 the constant θ lines for this impeller are highly skewed and cannot represent true quasi-orthogonals. Hence impellers having 60 deg. wrap angles are superior, when using the present techniques.

The suction and pressure surfaces of the blade were obtained by imposing blade thickness on the camber line. The suction and pressure surfaces of adjacent blades were obtained by adding pitch to the coordinates of first blade. The η values obtained for the suction surface of the first blade and the pressure surface of the adjacent blade were multiplied by the corresponding radius values to get the channel shape, which are shown in Fig. 5 . The given flow deflection is achieved through a shorter length of the channel for impellers possessing low wrap angle, giving rise to higher blade loading. The channel length increases with increase in wrap angle, leading to higher frictional loss which is counter-balanced by smaller diffusion loss due to moderate blade loading. The enlarged view of the channel at the shroud near the leading edge for impeller B-20 indicate that the blade angle turns from a given inlet angle to axial within a very short distance. Such a turning of blade is not good from the aerodynamic consideration. This is a typical characteristic of the analytic equation of Type-B at low wrap angles. Hence, at low wrap angles it is advantageous to use Type-A analytic equation.

The variation of elemental area of the channel along the meridional at hub and shroud would indicate imposed diffusion of relative flow. The elemental channel area was calculated by estimating the width and depth of the channel at a given location along the mer idional length. The width of the channel was calculated by finding the diameter of the smallest circle touching the suction and pressure surfaces of adjacent blades. As a first approximation, the depth of the channel was assumed to vary linearly from inlet to outlet with respect to meridional length. The inlet and outlet depth of the channel was assumed equal to the corresponding blade heights in the meridional plane. The local channel area was normalised with reference to inlet channel area and plotted against normalised meridional distance. The blade to blade channel area variation is shown in Fig. 6 . A sharp increase in area at the shroud near the leading edge for low wrap angle impeller B-20 would give large flow deceleration and hence high forward blade loading, which may cause a separation. A similar behaviour will be noticed in the results of flow analsis at the later stage. Hence for low wrap angles it is advantageous to use analytic equation of Type-A to define the impeller vane geometry. For high wrap angles either Type-A or Type-B analytic equation could be used to define the vane geometry, since both types of equation give almost identical channel area variation.

The variation of first derivative with respect to radius ratio for both types of analytic equations are shown in Fig. 7 . The variation of second derivative with respect to radius ratio for both types of analytic equations are shown in Fig. 8 . The first and second derivatives are functions of hub and shroud geometry in addition to vane geometry. The first and second derivatives of η with respect to radius for the impellers derived from both type of analytic equations are continuous and single valued throughout the region considered. The smooth variation of first and second derivatives with the same sign through out the region for impellers A-60 and B-60 ensure the flow to behave properly within the flow channel. Though the first and second derivatives vary smoothly for impeller A-20, they become negative within the mid channel before reaching again a positive value at outlet. The first and second derivatives for impeller B-20 sharply change in their values within a short distance from the impeller inlet, though the first derivative does not change sign. This is a consequence of sharp variation in blade camber line shape and passage as shown at the enlarged view in Fig. 5 .

7. THEORETICAL ANALYSIS

From previous Section, it was noticed that the impeller derived from analytic equation of Type-A is superior than that of the impeller derived from Type-B equation. The two impellers generated from Type-A analytic equation were theoretically analysed for comparative evaluation. The effect of wrap angle on the surface velocity distribution was investigated.

The two impellers designated by A-20 and A-60 were theoretically analysed for relative velocity flow field (9,10,11) and boundary layer (12) parameters along hub/shroud walls and suction and pressure surfaces of the blade.

The relative velocity distribution along hub and shroud surfaces for impeller A-20 and A-60 are shown in Fig. 9 . The local relative velocity is normalised with respect to inlet shroud relative velocity and it is plotted against normalised surface distance along hub and shroud walls. It is observed from this figure that for impeller A-60, the relative flow on the shroud wall decelerates gradually without much of adverse pressure gradient. The deceleration extends almost up to the impeller outlet. For Impeller A-20, on the shroud wall, there exists large flow deceleration in the initial portion, which could lead to early flow separation. For both the impellers A-20 and A-60 on the hub wall the relative flow exhibits almost same behaviour. The above observations indicate impellers possessing large wrap angles would give gradual flow deceleration on the hub/shroud walls, which is better from the aspect of boundary layer growth.

From the predicted relative velocity distribution the boundary layer analysis was carried out along the hub and shroud walls for impellers A-20 and A-60 assuming the flow as turbulent. The local skin friction coefficient is plotted against surface distance along hub and shroud walls (Figs 10a-b) . The local skin friction coefficient value at given normalised surface distance is higher on the hub wall as compared to that on the shroud wall due to larger surface distance. It is also observed from this figure that the local skin friction coefficient reaches a lowest value around m/m = 0.40 for impeller A-20 (Fig. 10a) and around m/m = 0.7 for impeller A-60

(Fig. 10b) . This indicate that the flow separates if at all, on the shroud wall for impeller A-20 earlier than that of impeller A-60. The estimated boundary layer growth at impeller outlet would be useful to estimate the effective blockage.

The relative velocity distribution along suction and pressure surfaces of the blade close to the shroud for impellers A-60 and A-20 are shown in Fig. 11 . Impeller A-20, which has low wrap angle exhibit steep pressure gradients on the pressure and suction surface of the blade within a short region from the leading edge. Which is attributed due to large initial blade curvature. The impeller A-60 has a large surface length with a smooth variation in channel area as seen in Section 5. This gives rise to uniform flow deceleration on the pressure surface with a small acceleration towards the trailing edge. On the suction surface of the blade the relative flow remains fairly constant. Fig. 12 indicate that, there is no significant difference in relative velocity distribution for impellers A-20 and A-60 near the hub wall. Both the impellers showed that, the relative flow on the suction and pressure surfaces initially decelerate and then accelerate.

The local blade loading coefficient was calculated from the relative velocity distribution. The blade loading coefficient was plotted against non-dimensional surface distance. Fig. 13 show the variation of local blade loading coefficient near shroud for impellers A-20 and A-60. It is observed from this figure that the area enclosed by the blade loading distribution curve for both the impellers are same, as they are designed for same duty. The maximum loading for impeller B-20 occurs around 40% of the surface distance. Whereas for impeller A-60 it occurs towards the trailing edge indicating that flow separation if any is delayed. This figure also indicates that the wrap angle has large influence on the blade loading. The blade loading distributionfor impellers A-20 and A-60 near hub wall as indicated in Fig. 14 shows a maximum loading towards the trailing edge. This indicates that the relative flow does not separate on the hub wall for both the impellers.

The theoretical analysis indicates that impeller with large wrap single is efficient from the consideration of blade loading as well as from boundary layer parameter. This flow analysis was mainly carried out to show the effect of wrap angle on impeller performance. The aerodynamic design of impeller need not go through such detailed analysis.

8. **CONCLUSIONS**

For a centrifugal impeller with 30 deg. backsweep different blade geometries were generated using newly evolved analytic equations. The wrap angle in the analytic equation served as an important parameter to control the diffusion along the channel length. Type-A and Type-B analytic equations for large wrap angles and Type-A analytic equation for small wrap angles are found suitable to define the blade geometry. For large axial lengths, an impeller having small wrap angle may be chosen with advantage. The point of inflexion in the first and second derivaties may be avoided by chosing the value of wrap angle slightly greater than the difference between inlet andf outlet blade angles. The evolved analytic functions make it convenient to define the impeller blade shape and allow its geometry to be controlled easily. The theoretical analysis indicates that impeller with large wrap angle is efficient from the consideration of blade loading as well as from boundary layer parameter.

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