ANTI-RESONANCE CONCEPT APPLIED TO ROTORS

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Introduction

Anti-resonance concept was first introduced by Lord-Rayleigh [1] with respect to string vibrations. With the development of Dynamic Anti-Resonant Vibration Isolator (DAVI) [2], the concept became popular with helicopter dynamists. Bartlett and Flannelly brought out the theory behind this principle [3]. Balasubramanian et.al applied this concept to Euler-Bernoulli beams [4]. The present work is the extension of the above, for rotor-dynamics.

Anti-Resonance Frequency Estimation

The methodology [3,4] involves eigen value extraction of an unsymmetric system. The i-jth anti-resonance frequency of a structure is defined as that forcing frequency, for which the response at the j-th location is zero, with the force at i-th location. Consider the generalised dynamic equation for a structure with n degrees of freedom, with force at i-th location.

$$\left[[K]-\omega^2[M]\right]\{x\}=\{F\} \quad (1)$$

$$\text{or} \quad [Z]\{x\}=\{F\} \quad (1a)$$

The equation 1a can be rewritten in a generalised form separating the forcing and response locations as shown in equation 1b, i.e.,

$$\begin{bmatrix} Z_{i,j_{i\neq k,j\neq l}} & Z_{i,j_{i\neq k,j=l}} \\ Z_{i,j_{i=k,j\neq l}} & Z_{i,j_{i=k,j=l}} \end{bmatrix} \begin{Bmatrix} x_{j_{j\neq l}} \\ x_{j_{j=l}} \end{Bmatrix} = \begin{Bmatrix} F_{i_{i\neq k}} \\ F_{i_{i=k}} \end{Bmatrix} \quad (1b)$$

By applying anti-resonance concept [3],

$$F_i = 0 \quad \text{for } i \neq k \quad (2a)$$

and

$$x_j = 0 \quad \text{for } j = l \quad (2b)$$

the i-th row and j-th column are removed as shown below:

$$\begin{bmatrix} Z_{1,1} & Z_{1,2} & \text{—} & Z_{1,j-1} & Z_{1,j+1} & \text{—} & Z_{1,n} \\ Z_{2,1} & Z_{2,2} & \text{—} & & \text{—} & & Z_{2,n} \\ Z_{3,1} & & \text{—} & & & & Z_{3,n} \\ \text{—} & & & & & & \text{—} \\ Z_{i-1,1} & Z_{i-1,2} & \text{—} & & & & Z_{i-1,n} \\ Z_{i+1,1} & Z_{i+1,2} & \text{—} & & & & Z_{i+1,n} \\ \text{—} & & & & & & \text{—} \\ Z_{n,1} & Z_{n,2} & \text{—} & & & & Z_{n,n} \end{bmatrix} \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ \text{—} \\ x_{j-1} \\ x_{j+1} \\ \text{—} \\ x_n \end{Bmatrix} = 0 \quad (3)$$

The resulting reduced [Z] matrix, i.e. [Z*] is solved for eigen values.

The eigen values of [Z*] are the i-j th anti-resonance frequencies of the structure. It should be noted here that [Z] is not symmetric. The method can be checked by directly solving the equation 1, varying the forcing frequency to obtain zero displacement at j-th location.

Finite element method has been used in this excercise. The element is a Euler-Bernoulli beam element with cubic displacement function, with displacement and slope as degrees of freedom at its two nodes.

Examples

Euler-Bernoulli Beams

T.S. Balasubramanian et.al have done extensive studies on the application of this concept to Euler-Bernoulli beams [4]. Cantilever, simply supported beams with steps as well with rotations have been studied. The results on cantilever beam (Fig. 1) have been compared with experiments, with good agreements. For the sake of continuity, results obtained on a cantilever beam are presented in Table-1.

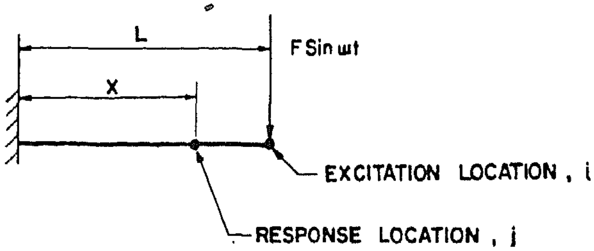

Fig.1 Cantilever Beam with Excitation and Response Locations

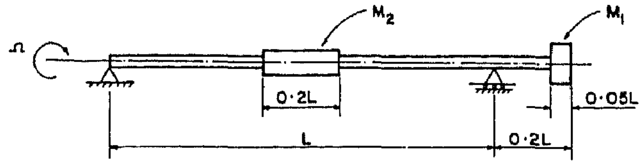

Fig.2 Beam Model of a Overhang Rotor with Central Mass (M_2)

Table -1 Anti Resonance Frequency Parameters λ_{ai} for a Cantilever Beam, excited at tip (Fig. 1)

	Anti-Resonance Frequency Parameters	
x/L	*Eigen Value Method*	*Force Responce Method*
1.0	15.4185	15.4186
0.9	17.4694	17.4696
0.8	21.1560	21.1532
0.7	28.4883	28.4885
0.6	42.2591	42.2592
0.5	62.5913	62.5923
0.4	97.1680	97.1681
0.3	172.891	172.891
0.2	391.886	391.886
0.1	2076.35	2076.34

Table - 2 Frequency Parameters λ_i and λ_{ai} for a Overhang Rotor (Fig. 2); ($M_1/mL=3$)

M_2/mL	*Natural Freq. Parameter*			*Anti-Res. Freq. Parameter*	
	Mode1	Mode2	Mode3	Mode1	Mode2
0.0	4.803	15.06	46.26	13.90	45.47
0.5	4.469	11.40	42.74	9.799	42.16
1.0	4.162	9.971	40.08	7.986	39.59
1.5	3.890	9.223	37.84	6.907	37.40
2.0	3.654	8.772	35.92	6.172	35.50
2.5	3.450	8.468	34.24	5.631	33.84
3.0	3.274	8.258	32.76	5.212	32.38
3.5	3.119	8.103	31.45	4.873	31.08
4.0	2.983	7.986	30.27	4.594	29.91

A Overhang Rotor

A typical overhang rotor system has been considered for the application of anti - resonance concept. The beam idealisation of the rotor system is given in Fig. 2. The anti-resonance frequency parameters λ_{ai} $[= (\omega_{ai}^2 EI/mL^4)^{1/2}]$ along with the natural frequency parameters λ_i $[= (\omega_i^2 EI/mL^4)^{1/2}]$ are presented in Table-2, for various M_2/mL. Here, E is the Young's modulus of the beam, I the geometric moment of inertia, L the length of the beam and m the mass per unit length of the beam. Suffix a refers to anti-resonance.

Let each unit of λ_i or λ_{ai} represent 1000 rpm in the real system. If there is no M_2, the critical speed will be around 4800 rpm. Let us assume that the rotor is rotating at a super-critical speed of 6172 rpm. Let anti-resonance principle be applied to this system, such that the impeller location of mass M_1 has zero displacement for the excitation at its own plane. Then by Table- 2, a mass of $M_2/mL = 2$ added to the rotor system will result in anti-resonance rpm of 6172 rpm. This will yield a smooth, vibration free rotor. The rotor will be insensitive to the vibration at the impeller plane.

Overhang Rotor with Flexible Supports

The system considered in the previous example is changed to flexible supports, removing the mass M_2 as shown in Fig.3. The variation of anti-resonance frequency for various spring parameters $\bar{K}_1$ and $\bar{K}_2$ $[\bar{K}_i = K_i L^3/EI]$ are presented in Table-3. Again here if 6600 is the rotor normal speed, anti-resonance will be achieved for $\bar{K}_1 = 10000$ and $\bar{K}_2 = 100$ (each unit of λ_i or λ_{ai} is equal to

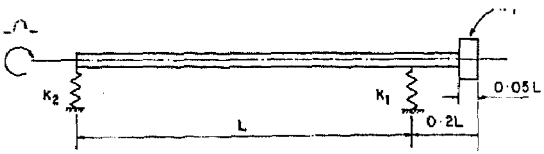

Fig. 3 Idealisation of a Overhang Rotor with Flexible Supports

Table - 3 Frequency Parameters, λ_i & λ_{ai} for a **Overhang Rotor on flexible Supports (Fig. 3)**; ($M_1/mL=3$)					
$\bar{K}_1,\bar{K}_2$	*Natural Freq. Parameters*			*Anti-Res. Freq. Parameters*	
	Mode1	Mode2	Mode3	Mode1	Mode2
Rigid	4.803	15.06	46.28	13.90	45.49
10000, 10000	3.753	11.76	31.91	6.686	30.46
10000, 100	3.670	11.73	31.88	6.605	30.44
10000, 1.0	1.599	11.35	31.46	5.085	29.97
10000, 0.1	0.562	11.28	31.63	4.747	29.87
1000, 1.0	1.601	10.59	26.36	4.772	26.04
1000, 10.0	3.115	10.79	26.62	5.786	26.32
500, 5.0	2.719	9.927	23.55	5.192	23.54
500, 2.0	2.079	9.851	23.44	4.757	23.43
500, 2.25	2.1632	9.860	23.45	4.801	23.43

1000 rpm). This concept can be effectively applied in autoclave blowers and other similar system using elastomer or flexible bearings.

Conclusions

Anti-resonance concept has been illustrated with respect to a overhang rotor. It is feasible to achieve the anti-resonance at the required plane by simple addition of mass or by flexible supports. This concept, applicable to supercritical rotors, can be effective and economical in reducing the vibration if used judiciously.

References

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