Some Spatial and Transform Domain Image Resolution Enhancement Algorithms – A Comparative Study

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Abstract— Image resolution enhancement algorithms, in spatial domain (viz., bicubic, data dependent triangulation and edge directed interpolation) and transform domain (viz., wavelet and DCT) were implemented and their performances were compared. It was concluded that discrete cosine transform (DCT) based image resolution enhancement algorithm performs better than the other methods. DCT based image resolution enhancement algorithm is very simple to implement and has the additional advantage of being able to produce high resolution images whose size need not necessarily be a multiple of two. DCT based video resolution enhancement algorithm was implemented and tested its performance in Simulink. It was concluded that this algorithm would be very suitable for video resolution enhancement.

II. SPATIAL DOMAIN METHODS

A. Bicubic Interpolation

This method is widely used everywhere, because it gives good results compared to the bilinear and nearest neighborhood interpolation algorithms at the cost of computation time. Bicubic method improves the model of the brightness function by approximating it locally by a bicubic polynomial surface. Sixteen neighboring points are used for interpolation. Bicubic interpolation preserves fine details in the image very well. The principle of bicubic interpolation is illustrated in Fig-1[2], where, \( H \) and \( L \) are high resolution (dotted grid) and low resolution (solid grid) images respectively.

\[
H(i', j') = \sum_{m=-1}^{2} \sum_{n=-1}^{2} L(i + m, j + n)R(m - dx)R(dy - n) \tag{1}
\]

where

\[
R(x) = \frac{1}{6} \left[ P(x + 2) - 4P(x + 1) + 6P(x) - 4P(x - 1) + P(x - 2) \right]
\]

and

\[
P(x) = \begin{cases} 
x & x > 0 \\ 
0 & x < 0
\end{cases}
\]

In general, imaging sensors used in the cockpit could have different resolutions. In order to produce an integrated image to be presented to the pilot, images from the different sensors should have the same resolution. For this purpose several image resolution enhancement algorithms have been proposed in literature [2-10]. In this report, both spatial (viz., bicubic, data dependent triangulation and edge directed interpolation) and transform domain (wavelet and DCT) image resolution enhancement algorithms are implemented and their performances compared.

B. Data Dependent Triangulation Method [3]

It is a recent and popular image interpolation algorithm in spatial domain. The advantage of this algorithm is edge-directed reconstruction, i.e., it matches the edges in the image and improves the reconstructed image. Consider a...
C. New Edge Directed Interpolation [4-6]

Classical linear interpolation methods fail to capture the fast developing statistics around edges and results in blurred edges and annoying artifacts in the interpolated image. Edge directed interpolation would preserve the edges. It estimates local covariance coefficients from a low-resolution image and these covariance estimates are used in the interpolation at a higher resolution based on the geometric duality between the low-resolution covariance and the high-resolution covariance.

\[ H_{2i+1,2j+1} = \sum_{k=0}^{1} \sum_{l=0}^{1} \alpha_{2k+1} X_{2i+1,2j+1} \]

where \( X_{2i,2j} = L_{i,j} \) and \( \alpha \) is linear interpolation coefficient.

The optimal minimum mean square error linear interpolation coefficients are computed as \( \alpha = R^{-1} \bar{r} \), where \( R = R_{ij}, 0 \leq k, l \leq 3 \) and \( \bar{r} = r_{k}, 0 \leq r_{k} \leq 3 \) are local covariance at the high resolution. These quantities can be estimated by local window of the low resolution image as \( R = \frac{1}{M^2} C^T C \) and \( \bar{r} = \frac{1}{M^2} C^T \tilde{y} \), where \( \tilde{y} = [y_1 \ldots y_k \ldots y_M] \) is the data vector containing the \( M \times M \) pixels inside the local window and \( C \) is a \( 4 \times M \) data matrix whose \( k^{th} \) column vector is the four nearest neighbors of \( y_k \) along the diagonal direction. From these, the linear interpolation coefficient can be \( \alpha = (C^T C)^{-1} C \bar{r} \).

III. TRANSFORM DOMAIN

A. Wavelet transform [7,8]

The most commonly used algorithms for image interpolation are linear and bi-linear interpolation for one- and two-dimensional data, respectively. These simple algorithms are usually sufficient for a rough estimate of the interpolated value, but can give erroneous results when high precision is needed. In general high resolution image is constructed from low resolution image directly by applying spatial domain interpolation methods. The statistical spatial domain interpolation methods will not yield good results, because the unknown pixel value is estimated by approximation of neighboring pixels. Wavelet transform decomposes the image into four sub band images such as LL, LH, HL and HH. Where LL contains the overall information, LH and HL contains the vertical and horizontal information and HH contains the diagonal information. Image resolution enhancement can be done by considering the low resolution image as LL and other sub band images as zeros of same size of low resolution image as shown in Fig-3, since the human eyes are very less sensitive to luminance than chrominance. High resolution image can be obtained by taking the inverse wavelet transform on these sub band images.

![Fig. 2. Illustration of triangulation in a four-pixel square](image_url)

![Fig. 3. Conventional wavelet domain image interpolation](image_url)

B. Discrete Cosine Transform [9,10]

The DCT based algorithms are generally developed for compressing the audios and videos, for reducing the storage capacity. DCT is applied for Image Resolution Enhancement, because it supports stretching the image with negligible loss. The important property of DCT is its efficient energy compaction and concentration of most of the power in the lower frequencies. This is important because the HVS (Human Visual Systems) is always very less sensitive to luminance than chrominance. DCT coefficients \( (L) \) are computed from low resolution image \( (L) \) and zeros are padded as shown in Fig-4. IDCT is performed on the resulting coefficients to get the high

![Diagram of DCT block](image_url)
1. Root mean square error (RMSE) [11]

\[ RMSE = \sqrt{\frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (H_r(i, j) - H(i,j))^2} \] (3)

Computed as the root mean square error of the corresponding pixels in the reference image \( H_r \) and the obtained high resolution image \( H \).

2. Percentage fit error (PFE) [12]

\[ PFE = \frac{\text{norm}(H_r - H)}{\text{norm}(H_r)} \times 100 \] (4)

where \( \text{norm} \) is the operator to compute the largest singular value. Computed as the norm of the difference between the corresponding pixels of reference image and obtained high resolution image to the norm of the reference image.

3. Mean absolute error (MAE) [11]

\[ MAE = \frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} |H_r(i, j) - H(i, j)| \] (5)

MAE is the mean absolute error of the corresponding pixels in reference and obtained high resolution images.

4. Peak signal to noise ratio (PSNR) [12]

\[ PSNR = 20 \log_{10} \left( \frac{L}{\sqrt{\frac{1}{MN} \sum_{i=1}^{M} \sum_{j=1}^{N} (H_r(i, j) - H(i, j))^2}} \right) \] (6)

where \( L \) in the number of gray levels in the image. Its value will be high when the high resolution and reference images are similar. Higher value implies better reconstruction of high resolution image.

5. Mutual information (MI) [13]

\[ MI = \sum_{i,j}^M \sum_{j=1}^N h_{H,H}(i,j) \log_2 \left( \frac{h_{H,H}(i,j)}{h_H(i,j) h_H(i,j)} \right) \] (7)

Larger value implies better image quality. Where \( h \) is the normalized histogram.

### V. PERFORMANCE EVALUATION METRICS

Performance of the above discussed interpolation algorithms are evaluated using the following metrics:

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>Bi-cubic</th>
<th>DDT</th>
<th>NEDI</th>
<th>Wavelet</th>
<th>DCT</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>7.11</td>
<td>11.75</td>
<td>8.58</td>
<td>8.21</td>
<td>5.28</td>
</tr>
<tr>
<td>PFE</td>
<td>7.07</td>
<td>11.69</td>
<td>8.53</td>
<td>8.16</td>
<td>5.25</td>
</tr>
<tr>
<td>MAE</td>
<td>3.26</td>
<td>5.56</td>
<td>4.14</td>
<td>3.83</td>
<td>2.6</td>
</tr>
<tr>
<td>PSNR</td>
<td>39.64</td>
<td>37.46</td>
<td>38.83</td>
<td>39.02</td>
<td>40.94</td>
</tr>
<tr>
<td>MI</td>
<td>1.35</td>
<td>1.27</td>
<td>1.31</td>
<td>1.32</td>
<td>1.40</td>
</tr>
<tr>
<td>Proc. time</td>
<td>0.656</td>
<td>0.719</td>
<td>115.72</td>
<td>0.110</td>
<td>0.281</td>
</tr>
</tbody>
</table>

### VI. RESULTS AND DISCUSSION

The original image (flower) of size 256x256 is shown in Fig-7a. It is taken as the reference image. Low resolution
Image is derived from this reference image by image decomposition using wavelet transform. The size of the low resolution image is 128x128 and is shown in Fig-7b. High resolution image is reconstructed using the above discussed interpolation methods and compared with reference image. High resolution image obtained from bicubic interpolation and corresponding error image are shown in Fig-8a and 8b respectively. Fig-9 and Fig-10 shows the high resolution image obtained from DDT and NEDI respectively. Similarly, Fig-11 and Fig-12 shows the high resolution image obtained from wavelet and DCT respectively. Performance evaluation metrics are provided in Table 1. The parameters shown in bold indicate best performance. From the results it is observed that for this data DCT performed better compared to the others. Edge directed interpolation algorithm preserves edges as shown in Fig-9, but it takes more computational time. The 1st frame of LR and HR video are shown in Fig-13. The root mean square error is shown in Fig-14. It is observed that the error is within ±3 pixels which shows the robustness of the algorithm and its capability to video resolution enhancement.

VII. CONCLUSION

Image resolution enhancement algorithms, in spatial (viz., bicubic, data dependent triangulation and edge directed interpolation) and transform domain (viz., wavelet and DCT), are presented and their performances compared. It is concluded that DCT based image resolution enhancement algorithm performs better compared to the other methods. DCT based image resolution enhancement algorithm is very simple to implement and has the additional advantage of being able to produce high resolution images whose size need not necessarily be a multiple of two. DCT based video resolution enhancement algorithm is implemented and performance is tested in Simulink. It is concluded that this algorithm would be very suitable for video resolution enhancement.

REFERENCES

Fig. 11. a) HR and b) error image by wavelets

Fig. 12. a) HR and b) error image by DCT

Fig. 13a. LR video (1st frame)  
Fig. 13b. HR video (1st frame)

Fig. 14. Root mean square error of video