

# Filtering and Fusion based Reconstruction of Angle of Attack

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## Abstract

This paper presents a scheme to provide analytical redundancy for angle of attack sensor which is an important feedback for flight control system of a high performance aircraft / missile and also for aircraft stall warning system. The filtering and fusion scheme for reconstruction of angle of attack utilizes the rigid body kinematic equations of the aircraft and the data from INS and air data computer in an extended Kalman filter. The scheme is implemented in MATLAB / SIMULINK environment and validated with both flight simulator data as well as flight data. The scheme provides an additional source of angle of attack which is reliable and reasonably accurate.

## Nomenclature

$u, v, w$	Velocity components along aircraft body axis
$p, q, r$	Aircraft angular rates
$A_x, A_y, A_z$	Aircraft linear acceleration along body axis
$N_z, N_y$	Aircraft normal and lateral acceleration
$\phi, \theta$	Aircraft roll and pitch attitude
$\alpha, \beta$	Aircraft angle of attack and angle of sideslip
$h, \bar{q}, \rho$	Aircraft altitude, dynamic pressure and air density
$C_{L0}, C_{L\alpha}$	Aircraft aerodynamic coefficients
$\alpha_{a1}$	Computed angle of attack from altitude rate and Euler angles

$\alpha_{a2}$	Computed angle of attack from normal acceleration, weight and aerodynamic derivatives
$\beta_a$	Computed angle of sideslip from $N_y - \beta - gain$ derivative
$g$	Acceleration due to gravity
$s, w_t$	Aircraft wing reference area and weight
$N_y - \beta - gain$	$N_y$ to $\beta$ gain derivative

subscript  $m$  denotes measurements  
ˆ denotes estimate

## 1 Introduction

Angle of attack (AOA) and angle of sideslip (AOS) provide information about the flow condition of the aircraft. It can contribute to flight safety by supporting avoidance of critical conditions like problematic stalling behavior of the aircraft and can be an aid concerning departure characteristics. For unstable aircraft, AOA information forms an important feedback signal in the flight control system (FCS). Also the stall warning system (SWS) in transport aircraft uses the AOA signal to compute the stall margin which is directly proportional to the ratio of true AOA to stall AOA.

Thus, both fighter as well transport aircraft require AOA information for all their mission requirements. In general, AOA and AOS are measured by vanes or pressure probes. They have to be placed outside the fuselage and at a proper location outside the flow field in order to provide correct indications of AOA. If vanes are used, they have to be mounted on nose booms on the fuselage and they are usually susceptible to damages particularly at high speed. Further, regular maintenance is required to ensure proper operation. The sensors, their installation and maintenance are a costly affair. It is mandatory to have sensor redundancy in aircraft for AOA measurement since it is an essential feedback signal for FCS. Hence, in this paper, a scheme to provide analytical redundancy for AOA is presented.

There are several approaches for determining the AOA and AOS based on data from the inertial navigation system (INS) instead of aerodynamic sensors. In this paper, an approach featuring estimation of AOA and AOS by filtering and fusion of INS data and air data computer outputs through the aircraft rigid body kinematics is presented. The goal is to achieve a redundant, reliable source of AOA and AOS information all the time with reasonable accuracy. An extended Kalman filter is implemented for filtering and fusion of INS and airdata sensor outputs and the scheme is implemented in MATLAB / SIMULINK environment and validated with light transport aircraft flight simulator data as well as real data from a fighter aircraft. The concept and scheme presented here can be easily extended and utilized for other aerospace guidance and testing systems.

## 2 AOA and AOS reconstruction scheme

AOA estimation has two important requirements, namely, speed and accuracy. Speed of estimation is a critical requirement for in-flight real time application for flight control usage of AOA as well as for pilot information. AOA and AOS can be estimated with high degree of accuracy from the measurements from inertial navigation system. Fusing the outputs of three accelerometers, three rate gyros and air data computer (total velocity, dynamic pressure and altitude), the AOA and AOS can be estimated through aircraft rigid body kinematic equations[1,2]. Thus it is purely an analytical method of reconstructing AOA and AOS without utilizing information from external devices like flow vanes. Kalman filter algorithm is used to estimate the AOA and AOS as it is recursive in nature and suitable for online data processing. The state and measurement equations of the filter are given below.

*State Equations:*

$$\begin{aligned}\dot{u} &= A_{x_m} - q_m w + r_m v - g \sin \theta \\ \dot{v} &= A_{y_m} - r_m u + p_m w + g \sin \phi \cos \theta \\ \dot{w} &= A_{z_m} - p_m v + q_m u + g \cos \phi \cos \theta \\ \dot{\phi} &= p_m + q_m \sin \phi \tan \theta + r_m \cos \phi \tan \theta \\ \dot{\theta} &= q_m \cos \phi - r_m \sin \phi \\ \dot{h} &= u \sin \theta - v \sin \phi \cos \theta - w \cos \phi \cos \theta\end{aligned}\quad (1)$$

*Measurement Equations:*

$$\begin{aligned}V_m &= \sqrt{u^2 + v^2 + w^2} \\ \bar{q}_m &= \frac{1}{2} \rho (u^2 + v^2 + w^2) \\ \phi_m &= \phi \\ \theta_m &= \theta \\ h_m &= h\end{aligned}$$

$$\begin{aligned}\alpha_{a1} &= \tan^{-1}\left(\frac{w}{u}\right) \\ \alpha_{a2} &= \tan^{-1}\left(\frac{w}{u}\right) \\ \beta_a &= \sin^{-1}\left(\frac{v}{\sqrt{u^2 + v^2 + w^2}}\right)\end{aligned}\quad (2)$$

where

$$\begin{aligned}\alpha_{a1} &= \sin^{-1}\left(\frac{\sin \theta_m}{\sqrt{\sin^2 \theta_m + \cos^2 \theta_m \cos^2 \phi_m}}\right) - \\ &\sin^{-1}\left(\frac{\frac{\dot{h}_m}{V_m} + \sin \beta_a \sin \phi_m \cos \theta_m}{\left(\sqrt{\sin^2 \theta_m + \cos^2 \theta_m \cos^2 \phi_m}\right) \cos \theta_m}\right)\end{aligned}\quad (3)$$

$$\alpha_{a2} = \frac{\left[\left(\left(\frac{g}{s}\right)(N_z + \cos \theta_m \cos \phi_m) \frac{w_l}{\bar{q}_m}\right) - C_{L0}\right]}{C_{L\alpha}}\quad (4)$$

$$\beta_a = -(N_y - \beta_{gain})N_y\quad (5)$$

$(N_y - \beta_{gain})$  is taken from the lookup table given as a function of dynamic pressure

In this model it is assumed that all the measurements are bias free and are at the aircraft centre of gravity. Appropriate changes in the kinematic model can be incorporated to estimate the bias in the measurements as augmented states.

Figure 1 shows the schematic diagram of the AOA and AOS estimation scheme. In Kalman filter, analytically obtained redundant AOA ( $\alpha_{a1}$  &  $\alpha_{a2}$ ) and redundant AOS ( $\beta_a$ ) are used as observations to estimate the relevant aircraft states. From the estimated (filtered) states, the AOA and AOS are computed as given below:

$$\begin{aligned}\hat{\alpha} &= \tan^{-1}\left(\frac{\hat{w}}{\hat{u}}\right) \\ \hat{\beta} &= \sin^{-1}\left(\frac{\hat{v}}{\sqrt{\hat{u}^2 + \hat{v}^2 + \hat{w}^2}}\right)\end{aligned}\quad (6)$$

where  $\hat{u}$ ,  $\hat{v}$  &  $\hat{w}$  are the estimated states from the filter.

## 3 MATLAB / SIMULINK implementation

The reconstruction of AOA and AOS is realized and studied in MATLAB / SIMULINK environment

### 3.1 Analytical AOA and AOS

Analytical redundant AOA and AOS are obtained from INS and airdata computer outputs using equations (3, 4

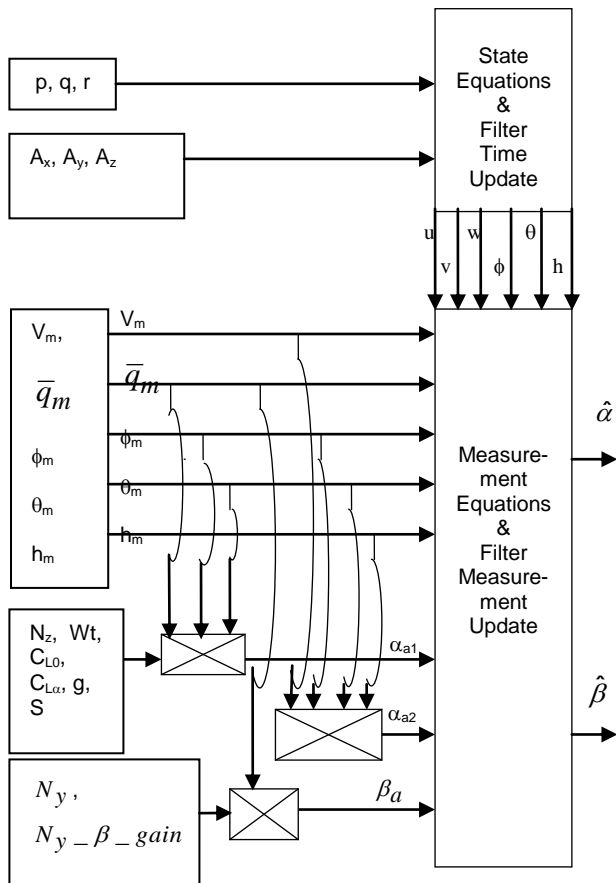


Figure 1: AOA and AOS reconstruction scheme

& 5) [3]. These equations are implemented in SIMULINK and the output of this implementation has been independently checked with respective reference values (true values in case of flight simulator data and vane angles in case of real data). It has been noted in the reference [3] and confirmed here that the redundant AOA derived from altitude rate and Euler angles (equation 3) are very accurate wherever there is no gust or turbulence. However in the presence of vertical gust/turbulence, the redundant AOA derived using  $N_z$ , weight,  $C_{L0}$  and  $C_{L\alpha}$  (equation 4) is relatively better. This is because sufficient information of vertical gust/turbulence is captured in  $N_z$  measurement.

### 3.3 Kalman Filter

Kalman filter algorithm is introduced into the SIMULINK block as S-function. The filter model consists of two components, a nonlinear kinematic state model and a nonlinear measurement model. The state model predicts the states  $(u, v, w, \phi, \theta, h)$  from one instant of time to the next instant based on the input data from the accelerometers and rate gyros. The measurement model corrects this prediction with the reference measurements obtained from the air data sensors and analytically computed AOA and AOS (equations 2, 3

and 4). The implementation details of Kalman filter are given in reference [4].

The filter is manually tuned and its performance is tested by plotting and checking:

1. Estimated AOA and AOS with true AOA and AOS (vane outputs in case of flight data).
2. Estimated filter model outputs with measurements
3. Filter residual with bounds  $\pm \sqrt{R_e}$  where  $R_e$  is the innovation variance
4. Auto correlation of the residual with bounds  $\pm \frac{1.96}{\sqrt{N}}$  where N is the no. of samples, (to confirm the whiteness of the residual)

## 4 Validation Results

AOA and AOS estimation scheme is initially validated with light transport aircraft flight simulator data. Same set of simulated data used for evaluation of INS based AOA estimation scheme for stall warning system by autopilot team [3] is being used here for validation of the present scheme. Two sets of data are considered one with only elevator input and the other with elevator input and with vertical gust of 5 m/s.

The results obtained from the first set of simulated data (i.e. without gust) are shown in figures 2(a) to 2(c). Figure 2(a) shows the estimated outputs of AOA and AOS from the filter compared with simulated true AOA and AOS along with the analytically computed AOA and AOS which were used as observables in the Kalman filter. The estimated signals compare well with the true AOA and AOS signals. Fig. 2b shows the residuals of all the 8 observations with bounds and Fig. 2c shows the autocorrelation function with bounds. It is clear that all the residuals are well within their theoretical bounds and the autocorrelation function shows that the residuals satisfy the whiteness test. Thus, the proposed scheme indicates satisfactory performance when the aircraft flight is in calm air.

Figure 3 shows the AOA and AOS estimates from the light transport aircraft simulator data generated with elevator and vertical gust. During the presence of vertical gust, since  $\alpha_{a2}$  (obtained from  $N_z$ , weight,

$C_{L0}$  and  $C_{L\alpha}$ ) is relatively better compared to  $\alpha_{a1}$  (obtained from attitudes and altitude rate), the measurement noise covariance of these observations in the Kalman filter are rescheduled giving more emphasis to  $\alpha_{a2}$  observation (with lower measurement noise covariance). Thus whenever vertical gust is present it is possible to reconstruct AOA with reasonable accuracy without modeling of gust using this scheme. However, this method would require some kind of an indication of the presence of gust.

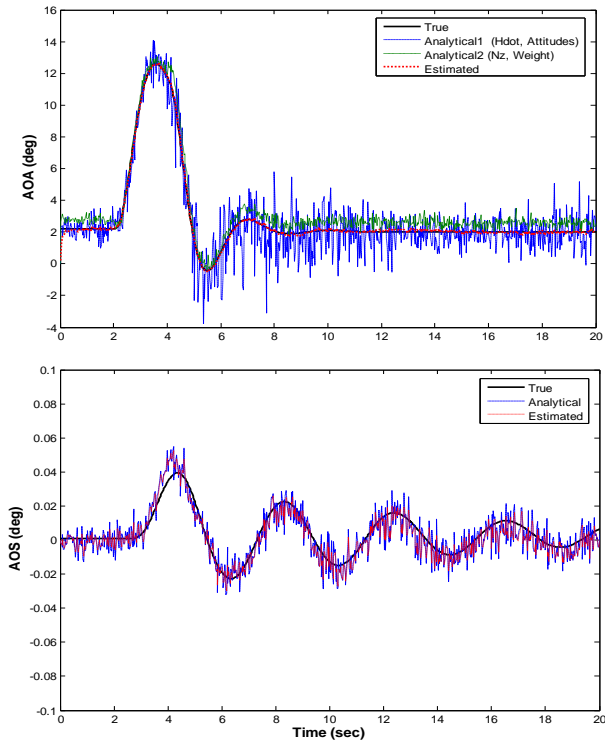


Figure 2(a): Reconstructed AOA and AOS trajectories

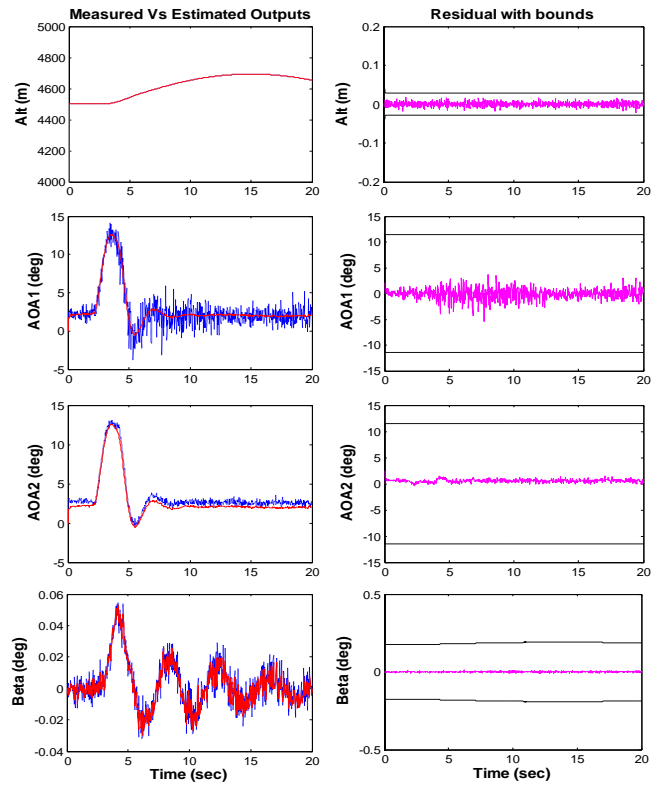


Figure 2(b): Continued ...

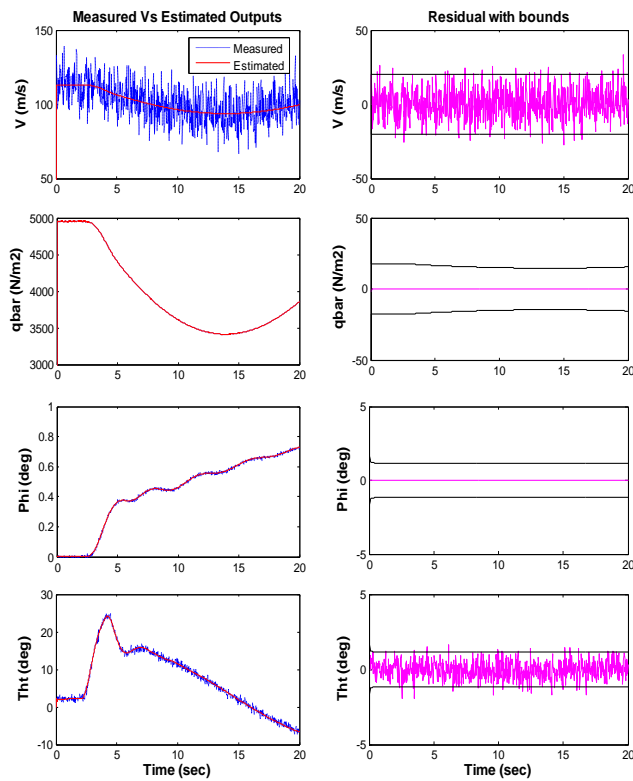


Figure 2(b): Kalman filter performance check (Filter residuals with bounds)

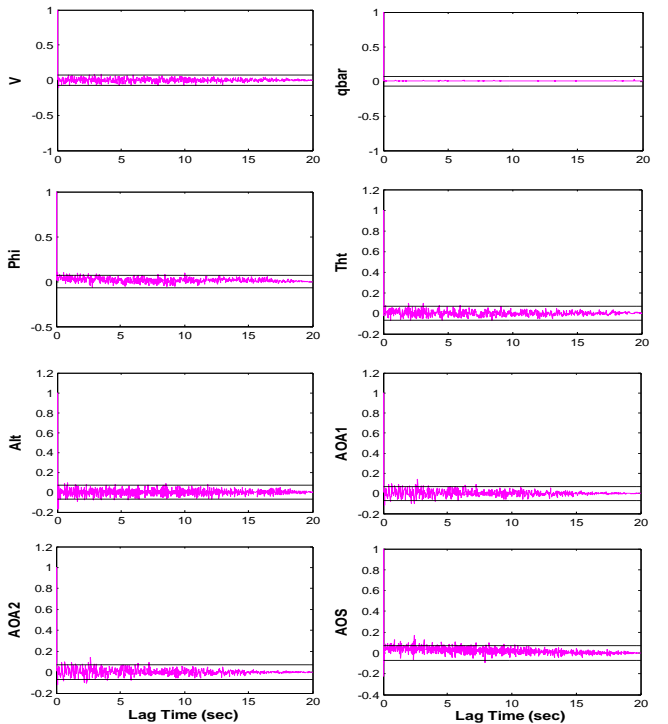


Figure 2(c): Kalman filter performance check (Autocorrelation of residual for whiteness test)

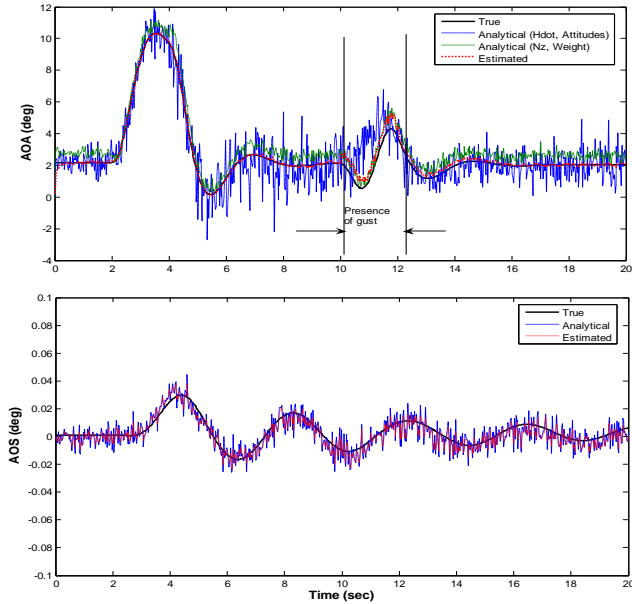


Figure 3: Reconstructed AOA and AOS trajectories in the presence of vertical gust

Figures 4 and 5 show the reconstruction of AOA from the real (flight) data of a fighter aircraft from different maneuvers. Here the vane output which is found to be accurate (after correcting by kinematic consistency check) is taken as reference. Results indicate that the proposed algorithm successfully reconstruct the AOA despite noisy data. It can be seen that the reconstructed AOA is reasonable accurate even when one of the analytical redundant observation ( $\alpha_{a2}$ ) not being very accurate.

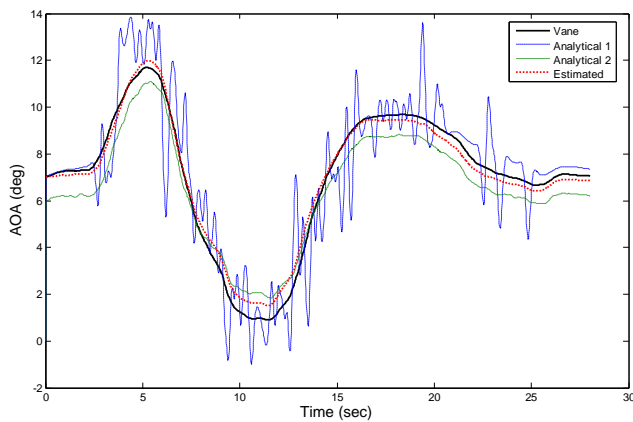


Figure 4: Reconstructed AOA from roller coaster maneuver of fighter aircraft

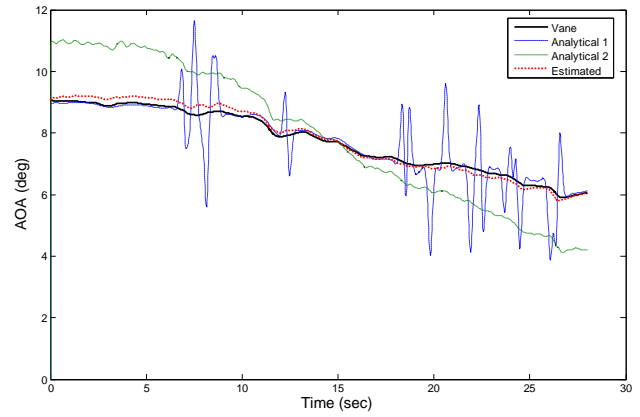


Figure 5: Reconstructed AOA from level acceleration maneuver of fighter aircraft

## 5 Concluding Remarks

A scheme based on filtering and fusion using an extended Kalman filter to provide analytical redundancy for AOA and AOS sensors in an aircraft is presented. The filtering and fusion scheme for reconstruction of AOA and AOS utilizes the rigid body kinematic equations of the aircraft and the data from INS and air data computer. The scheme is validated with aircraft simulator data as well as flight data. The results obtained indicate that the AOA and AOS are reconstructed with reasonable accuracy. The scheme can be used to reconstruct AOA and AOS which are critical parameters during flight emergency situations like landing and also for stall warning.

## 6 References

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