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FINITE ELEMENT ANALYSIS FOR THERMAL BUCKLING BEHAVIOUR IN FUNCTIONALLY GRADED PLATES WITH CUT-OUTS

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Abstract: Finite element formulation for the thermal buckling of moderately thick rectangular functionally graded material (FGM) plates is developed. This is based on the first order shear deformation theory (FSDT). One dimensional heat conduction equation is employed to represent the temperature distribution across thickness of the FGM plate. Material properties of the plate are considered to be function of temperature. It is assumed that the material properties of the FGM plate vary as a power function along the plate thickness. Finite element code is developed and computation of critical thermal buckling temperature of the FGM plates is carried out. The developed finite element computer code is validated with the results available in the literature. Further, finite element analysis is carried out to determine the thermal buckling of rectangular FGM plates with circular cutout. In the present work effects of (i) plate aspect ratio, (ii) plate thickness to side ratio, (iii) power index '*k*' (iv) size of the cutout and (v) the three different boundary conditions on the critical buckling temperature are being investigated.

1. INTRODUCTION

Functionally Graded Materials (FGMs) are those in which the volume fractions of two or more materials are gradually varied as a function of position and desired distribution of material properties are obtained. For example composition of ceramic and metal. By gradually varying the volume fraction of the constituent materials in the FGMs, stronger and tougher materials are achieved. These special properties make the functionally graded materials to be preferred for various aerospace, mechanical and medical applications. Further, applications of the FGMs have been widely observed in the high temperature environments, including thermal shock. Thermal analyses of FGM structures are reported by Ravichandran (1995), Praveen and Reddy (1998), Pradhan *et.al.* (2000), Cho and Oden (2000), Woo and Maguid (2001), Kim and Noda (2002), Tsukamoto (2003), Yang *et al.* (2003), Lanhe (2004), Pradhan (2005) and Sladeka *et al*. (2005). Buckling analyses are being reported by Yang *et. al.* (2005) and Huang and Shen (2006). Thermal buckling of composite materials are being reported by Chang and Shiao (1990), Chen (1991), Prabhu and Dhanraj (1994) and Avei *et al.* (2005). Pradhan *et.al.* (1995), Reddy (2003), Cook *et al* (2003) and Sengupta *et.al.* (2005) have reported various applications of finite element analysis.

However, research reports in the area of thermal buckling of FGM plates with cutouts are limited in the literature. In the present work, based on first order shear deformation theory (FSDT) finite element code is developed. The computer code is validated with results available in the literature. Further, computation is being carried out for thermal buckling of FGM plates with cutouts. Material properties are considered to be function of temperature. Effects of plate aspect ratio, plate thickness to side ratio, power index '*k*', size of the cutout and three different types of boundary conditions on the critical buckling temperature are studied.

2. MATHEMATICAL FORMULATION

A rectangular plate made of a mixture of metal and ceramic is considered. The material properties are gradually varied from bottom surface pure metal to top surface pure ceramic. An exploded view with granules of section of the plate is shown in Fig. 1. This depicts the gradation from pure metal to pure ceramic across the thickness. Effective material properties P_f , like Young's modulus ' E ' and thermal expansion coefficient ' α ', are expressed as in Pradhan (2005),

$$P_f = P_c V_c + P_m V_m \qquad (1)$$

where, V_c denotes the volume fraction of the ceramic. Subscript c and m represents ceramics and metal, respectively. V_c is expressed as

$$V_c = \left(\frac{2z+h}{2h}\right)^k \quad \text{where,} \quad -h/2 \le z \le h/2 \qquad (2)$$

From eq.(1) and eq.(2) the modulus of elasticity 'E', the coefficient of thermal expansion 'α', and the thermal conductivity 'K' of the FGM are expressed as P_f,

$$P_f = (P_c - P_m)P_c + P_m \qquad (3)$$

Poisson's ratio 'ν' is assumed to be 0.3 for both the materials. In actual case, material properties are dependent on the temperature. Change in temperature does affect the strength and stiffness of the FGM plate. Thus it is necessary to include the effects of temperature in the thermal buckling analysis of the FGM plate. Material properties as a function of temperature are employed in the analysis. Material properties can be expressed as a nonlinear function of temperature as mentioned in Pradhan (2005),

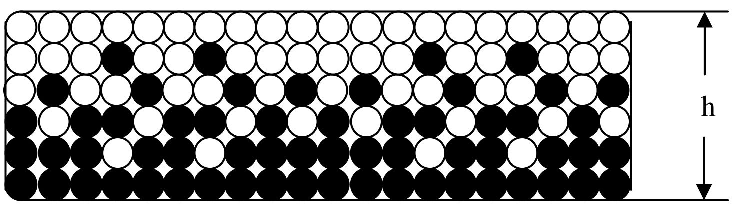

Fig. 1. Schematic of gradation of metal and ceramic across the plate thickness

$$P_f = P_0(P_{-1}T^{-1} + 1 + P_1T^1 + P_2T^2 + P_3T^3) \qquad (4)$$

Poisson's ratio 'ν' depends weakly on temperature change and is assumed to be constant. The temperature change across the thickness of the FGM plate is governed by one dimensional Fourier heat conduction equation as mentioned by Wu Lanhe (2004). In the present analysis of FGM plate first-order shear deformation theory (FSDT) is employed. Detail mathematical derivations are presented in Reddy (1997). Thermal force and moments are given as,

$$[N_t] = \int_{-h/2}^{h/2} Q_{ij} \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ 0 \end{Bmatrix} dz \quad \text{and} \quad [M_t] = \int_{-h/2}^{h/2} Q_{ij} \begin{Bmatrix} \alpha\Delta T \\ \alpha\Delta T \\ 0 \end{Bmatrix} zdz \qquad (5)$$

where, ΔT is the temperature difference between the temperatures of the undeformed plate and stress free plate. Eight noded isoparametric element is employed for the analysis. Total potential energy is the sum of the strain energy and the work done by the in-plane loading.

$$\pi = \pi_1 + \pi_2 \qquad (6)$$

where π_1 is the strain energy and π_2 is work done by the in-plane loading due to temperature change. Strain energy is expressed as

$$\pi_1 = \frac{1}{2}\iiint_V \{\varepsilon\}\{\sigma\}dV = \frac{1}{2}\iint_R [N_x\varepsilon_x + N_y\varepsilon_y + N_{xy}\varepsilon_{xy} + Q_x\varepsilon_{xz} + Q_y\varepsilon_{yz}]dxdy \qquad (7)$$

work done by inplane thermal loading is written as

$$\pi_2 = \frac{1}{2}\int_\Omega \left[N_x\left(\frac{\partial w}{\partial x}\right)^2 + N_y\left(\frac{\partial w}{\partial y}\right)^2 + 2N_{xy}\left(\frac{\partial w}{\partial x}\right)\left(\frac{\partial w}{\partial y}\right)\right] dA \qquad (8)$$

For the critical buckling state corresponding to second variation of total potential energy is equated to zero *i.e* $\frac{\partial^2 \pi}{\partial q^2} = 0$ which yields

$$\left|[K_0] + \lambda [K_g]\right| = 0 \qquad (9)$$

where $[K_0]$ and $[K_g]$ represent global stiffness and global geometric matrices, respectively. λ is determined by solving the above eigenvalue problem. The product of λ and the temperature difference ΔT corresponds to critical buckling temperature T_{cr} of the FGM plate.

3. VALIDATION

Developed finite element computer code is first validated for an isotropic plate as mentioned in Boley and Weiner (1960) and Chen (1991). As a particular case of FGM material, when the volume fraction index k is considered to be 0, the FGM material behaves exactly as isotropic and homogeneous material. Material properties listed in Chen (1991) are employed $E_1 / E_2 = 1$, a/t = 100, $\nu = 0.3, \alpha = 1 \times 10^{-6} (/^0 C)$. A refined finite element (eight node quadrilateral) mesh of two thousand elements is employed for the analysis. Critical buckling temperatures are being computed. Non-dimensional buckling temperature (T_{cr}) is written as T_{cr} = $\lambda \Delta T \times \alpha \times 1 \times 10^4$. Table.1 shows the comparison of non-dimensional critical buckling temperature of isotropic plate subjected to temperature distribution with those listed in Boley and Weiner (1960) and Chen (1991).

Table 1. Non-dimensional critical buckling temperature for isotropic plate

		0.5	1.0	1.50	2.0	2.50	3.0
Boley and Weiner 1960	0.686	0.808	1.283	2.073	3.179	4.599	6.332
Chen (1991)	0.691	0.814	1.319	2.101	3.191	4.601	6.330
Present Analysis	0.691	0.808	1.315	2.085	3.205	4.612	6.333

From Table.1, it is observed that the present computed results of critical temperatures agree with those reported in the literature Boley and Weiner (1960) and Chen (1991). Thus the developed finite element code does predict the critical buckling temperature with acceptable accuracy.

4. RESULTS AND DISCUSSION

To study the thermal buckling of a rectangular FGM plate with circular cutout is being carried out. Developed finite element code is employed for the analysis. For the symmetric cases one quarter of the plate is analyzed with appropriate boundary conditions. A typical one quarter plate finite element mesh of 1033 elements and 320 quadrilateral eight noded elements is depicted in Fig 2. However, a refined finite element mesh of three thousand such elements and full mesh is employed for the analysis. The FGM plate is considered to be made of silicon nitride and stainless steel. Mechanical properties of these materials and temperature dependent constants are listed in Table 2. In the analysis material properties of the constituent materials are considering to be nonlinear function of temperature as mentioned in eq (4).

Table 2. Material constants of Silicon Nitride and Stainless Steel, Shen and Noda (2005)

Material	Properties	P_o	P_{-1}	P_1	P_2	P_3
Silicon nitride	E_c	348.43e+9	0	-3.07e-4	2.16e-7	-8.946e-11
	α_c	5.8723e-6	0	9.095e-4	0	0
	K_c	13.723	0	0	0	0
Stainless steel	E_m	201.04e+9	0	3.079e-4	-6.534e-7	0
	α_m	12.33e-6	0	8.086e-4	0	0
	K_m	15.379	0	0	0	0

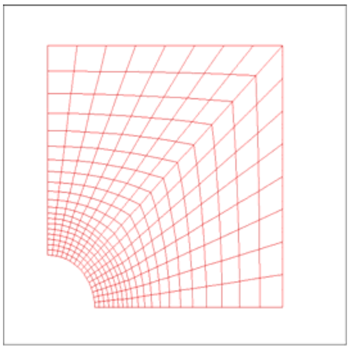

Fig. 2. Finite element mesh of rectangular FGM plate with circular cutout.

Three different boundary conditions are considered for the computation of the critical buckling temperatures of the FGM plate with cutout. *Viz.* (i) BC1, all the sides are simply supported (SSSS), (ii) BC2, all the sides are clamped and BC3 two opposite sides are clamped and other two sides are (iii) simply supported (CCSS). Thus following degrees of freedoms are restrained for the three boundary conditions. Critical thermal buckling load of the rectangular FGM plate with circular cutouts has been computed for various cases. The FGM plate subjected to uniform temperature loading is considered. Fig.3 shows the critical buckling temperatures of FGM plate for various thickness to side (h/a) ratios and different boundary conditions. In this figure one can note that buckling temperature increases gradually with increase in cutout size in case of all edges simply supported case BC1 (SSSS) and clamped-simply supported BC3 (CCSS) case. But in case of all edge clamped BC2 (CCCC), critical buckling temperature increases rapidly as cutout size increases. Buckling temperature also increases with increase of h/a ratio. It also shows that buckling temperature is highest for all sides clamped BC2 (CCCC) case. Fig.4 shows the effect of volume fraction index variation on critical buckling temperature. It shows that, buckling temperature decreases with increase in volume fraction.

There is rapid decrease in buckling temperature as power index *k* increase from 0 to 2. When *k* is greater than 2 buckling temperature almost remains constant for BC1 and BC3 cases. Metal content in the plate increases as power index increases. This leads to decrease in buckling temperature. Figs. 5-6 show the effect of temperature dependent material properties on critical buckling temperature. Here one could note that temperature dependent material properties causes substantial (around 25 per cent) decrease in critical buckling temperature of the FGM plate for BC1, BC2 and BC3 cases. This is because of the fact that increase of temperature causes reduction in stiffness of the plate. These results obtained for FGM plate with cutout are similar to the thermal buckling results of isotropic and composite plates with cutout by Chang and Shiao (1990).

5. CONCLUSIONS

Based on first order shear deformation theory finite element code is developed. The developed computer code is validated with results for thermal buckling of isotropic and FGM plates. Results obtained from present finite element analysis do agree with those reported in the literature. Developed finite element code is employed to compute the critical buckling loads of rectangular FGM plate with circular cutout. FGM material properties are considered to be temperature dependent. Effects of (i) plate aspect ratio, (ii) plate thickness to side ratio, (iii) power index '*k*' and (iv) size of the cutout and (v) the three different boundary conditions on the critical buckling temperature are investigated. Critical buckling temperature increases with increase in radius of the cutout in the rectangular FGM plate. Critical buckling temperature of the FGM plates increases with increase in thickness to span ratio. Critical buckling temperatures of the FGM plates decrease with increase in power law index '*k*'. Critical buckling temperatures of the FGM plates are decreased when material properties are considered to be function of temperature as compared to the results obtained where material properties are assumed to be independent of temperature. Critical buckling temperatures for BC1 (SSSS) boundary condition are larger than the corresponding temperatures obtained for BC2 (CCCC) and BC3 (CCSS) boundary conditions.

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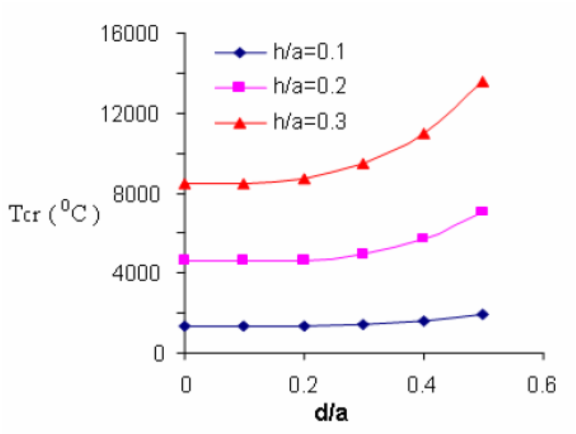
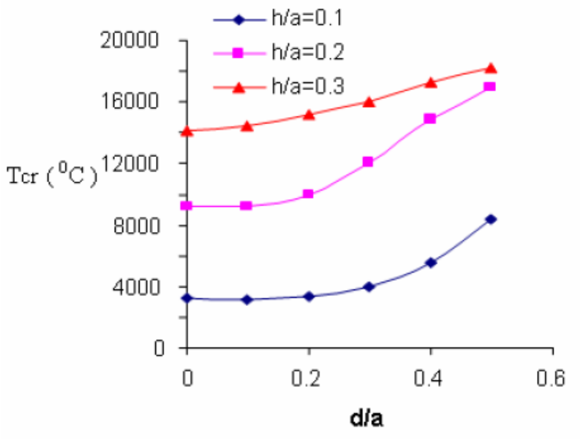
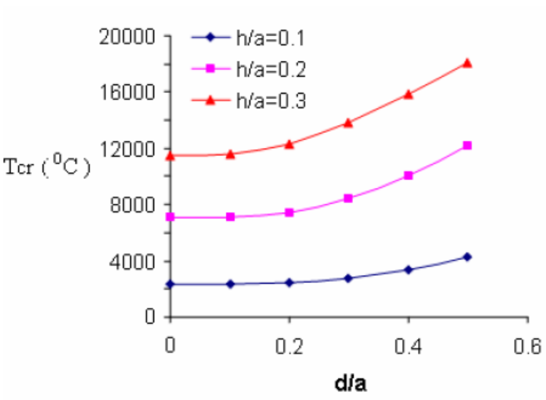


(a)

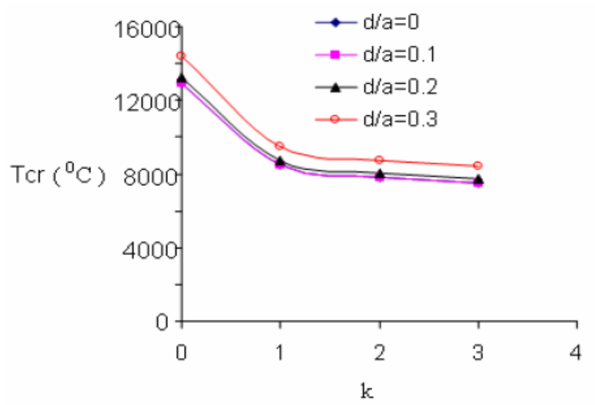


(b)

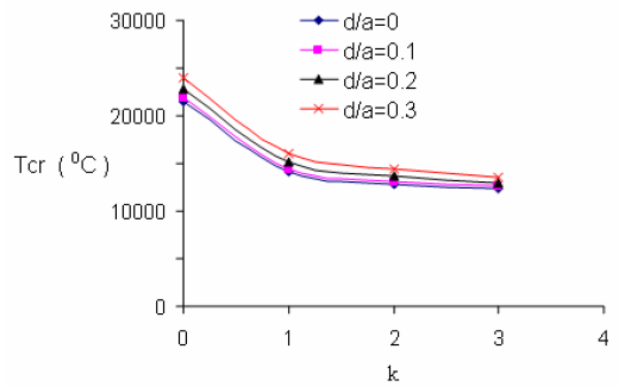


(c)

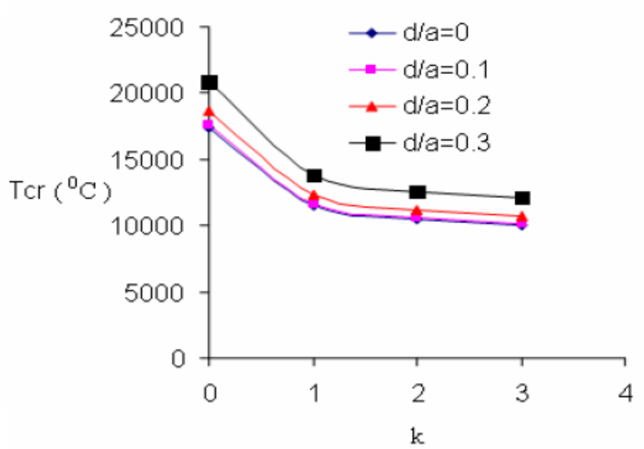
Fig.3. Critical buckling temperature *vs* the sizes of the cutout in the FGM plate, (a/b=1,k=1) for (a) BC1 (b) BC2, (c) BC3 boundary conditions



(a)



(b)



(c)

Fig.4. Critical buckling temperature *vs* power index '*k*' of the FGM plate, (a/b=1, h/a=0.3) (a) BC1 (b) BC2 and (c) BC3 boundary conditions