AEREOELASTIC RANDOM GUST RESPONSE ANALYSIS OF AIRCRAFT STRUCTURES IN TIME DOMAIN USING SIMULATIONS

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ABSTRACT: This paper presents a time domain method for determining the dynamic aeroelastic response of aircraft lifting surfaces under random gust loads. The random response analysis is carried out using a combination of Newmark’s technique and Monte Carlo Simulations (MCS). Newmark’s direct integration method has been employed for solving the differential equations of motion in time domain. The proposed method is based on aerodynamic strip theory. The unsteady aerodynamic forces on the wing are calculated from the theory of two-dimensional thin airfoils in non-uniform motion. The theory takes proper account of ‘unsteadiness’ by using the Kussner’s and Wagner’s unsteady lift functions while calculating the aerodynamic forces. The random gust load is modelled as a stationary random process and specified as an ensemble of velocity time histories generated using MCS. The proposed method is demonstrated by considering the case of an aircraft, which is free in vertical translation with wing being elastic in bending and restrained against twisting. The studies reported in this paper demonstrate the advantage of combining Newmark’s technique with MCS in the realistic estimation of random dynamic response of aircraft structures subjected to random gust loads. The results of such studies will be useful to carryout reliability analysis and safety assessment of aircraft structures.

1. INTRODUCTION
The rapidly varying aerodynamic loads due to gust play a vital role in the design of lifting surfaces, such as, wings, vertical tails, and horizontal tails of aerospace structures. The aerodynamic gust loads acting on the structure vary randomly with space and time and therefore should be modelled using suitable stochastic models. Hence, the study of aerodynamic gust response of an aerospace structure in flight is one of the most important random dynamic problems. Furthermore, aeroelastic effects resulting due to the interaction between aerodynamic loads and structural deformations may have a significant influence on the magnitude and distribution of aerodynamic loads due to gust \([1, 2]\). Thus, the aerodynamic gust response analysis of lifting surfaces of aerospace structures requires attention to two main factors, namely, the stochastic character of the aerodynamic loads and the aeroelastic effects. In the present work, an attempt is made to propose a time domain method for determining the dynamic aeroelastic response of aircraft lifting surfaces under random gust loads. The random response analysis is carried out using a combination of Newmark’s technique and Monte Carlo Simulations (MCS). The proposed method is demonstrated by considering the case of an aircraft, which is free in vertical translation with wing being elastic in bending and restrained against twisting.

2. AEREOELASTIC GUST RESPONSE ANALYSIS
Newmark’s direct integration method has been employed for solving the differential equations of motion in time domain, which is a departure from the usual frequency domain approach based on modal analysis \([3, 4]\). The proposed method is based on aerodynamic strip theory, but compressibility and aspect ratio corrections may be included, if desired. The unsteady aerodynamic forces on the wing are calculated from the theory of two-dimensional thin airfoils in non-uniform motion. The theory takes proper account of ‘unsteadiness’ by using the Kussner’s and Wagner’s unsteady lift functions while calculating the aerodynamic forces. The proposed method is formulated by considering the case of an aircraft, which is free in vertical translation with wing being elastic in bending and restrained against twisting. Also, the effect of the pitching motion and tail effects are neglected. It is assumed that the airplane velocity is sufficiently low so that the air may be assumed incompressible. The variation in the forward velocity of the airplane is neglected as it traverses the gust. The wing is assumed to be slender with arbitrary mass and stiffness. The wing has been modelled using the Euler-Bernoulli’s beam elements.
3. MONTE CARLO SIMULATIONS
Monte Carlo Simulation technique is a powerful tool that can provide solutions for certain class of problems in stochastic mechanics involving nonlinearity, system stochasticity, random excitations and large variations in uncertain parameters. The major advantage of MCS is that realistic solutions can be obtained for problems for which deterministic solution (either analytical or numerical) is known. One of the important steps in MCS technique, as applied to random response analysis of structures, is the generation of sample functions of the random process representing the random excitation. The generated sample functions should describe the probabilistic characteristics of the corresponding random process as closely as possible. From literature it is noted that various methods, such as spectral methods and ARMA methods, are available to generate the sample functions. The sample functions of the stationary process for random gust velocity, consistent with the specified spectral density function used for the present study, have been generated using the spectral method given by Grigoriu \[5\].

4. PROPOSED APPROACH
In the approach proposed in this paper, Newmark’s method is used for the estimation of gust response of the structure. The random gust load is modeled as a stationary random process and specified as an ensemble of velocity time histories. For each sample velocity time history, deterministic response has been evaluated using Newmark’s. Thus, the random dynamic response analysis involves the use of a hybrid technique consisting of Newmark’s method and MCS as demonstrated by the author earlier in the context of stochastic seismic problems \[6\]. The dimensionless time histories of bending moment at the root of the wing are obtained as the response output in each simulation cycle. The ensemble of bending moment time histories obtained is processed further to obtain the temporal characteristics such as the time histories of mean and variance of random the response at the chosen point.

5. EXAMPLE PROBLEM
The aim of this example taken from Bisplinghoff et al. \[1\] is to calculate the aeroelastic dynamic bending moment at the wing root of the jet transport airplane flying at a speed \(U = 475\) mph (Mach number, \(M = 0.62\)) due to a random gust. It is assumed that the aerodynamic forces due to the motion are defined satisfactorily by quasi-steady aerodynamic theory. The planform of the wing is shown in Figs. 1 and 2. The curves of bending rigidity \(EI\), torsional rigidity \(GJ\) and shearing rigidity \(GK\) have been plotted in Fig 3. The concentrated mass system of the wing is shown in Fig 4. Semi-span of the elastic wing, \(L\) is 41.67 ft. Total surface area of full span of the wing, \(S_T\) is 1128.47 ft². Reference half-chord of the wing, \(b_h = \frac{S_T}{4L}\) is 6.771 ft. Total mass of the airplane, \(m\) is 2606.1 Slugs; Mass density of the air, \(\rho\) is taken as 0.0765 lb/ft³. The chord variation of the wing (in ft) along the span wise direction is given by the expression, \(c = 18.75 \left(1 - \frac{y}{75}\right)\).

Modelling
The full span of the unrestrained \((i.e.,\, free-free)\) wing is considered and is divided into eleven, not necessarily equal sections, with a station point at the middle of each section. The Fig. 2 shows the division of semi-span of the wing into 5 sections, (the section representing the fuselage mass is not shown) which has been extended to full span of the wing in the present analysis. The station 6 is located at the center line of the fuselage. Here, the root of the wing is assumed to be station 6. The ten sections are divided in such a manner that most stations fall at concentrated mass locations or at points which represent the average of distributed masses, stations 1 and 11 being nearest to the tip of the wing. The total mass within each section is assumed to be concentrated at the respective station points. The surface areas of all the eleven wing sections are calculated using the trapezoidal method. The wing is modeled structurally with 10 equivalent Euler-Bernoulli’s beam elements. The 11 concentrated masses at each station represent the 11 nodal masses at the extreme ends of each of 10 elements (Fig. 5). The static condensation technique has been used to reduce the order of the assembled stiffness matrix by eliminating the rotational degrees of freedom of the Euler-Bernoulli’s beam elements. Quasi-steady aerodynamic strip theory approximation is assumed in the present analysis.
Solution and Results

The random dynamic response analysis involves the use of a hybrid technique consisting of Newmark’s method and MCS as discussed in section 4. The airplane is assumed to be flying through a random gust of mean velocity \( U \) and variance \( \omega_0^2 = 15\pi \) (ft/sec)\(^2\). An ensemble of 100 synthetically generated sample stationary velocity time histories consistent with the power spectral density function given by Press and Mazelsky \(^7\) is used to model the random gust excitation while carrying out MCS. The plots of the power spectral density function used and a typical realization of the velocity time history generated are shown in Figs. 6 and 7.

For each sample gust velocity time history, deterministic response has been obtained using Newmark’s method. The dimensionless time history of root bending moment (RBM) is obtained as output in each simulation cycle. The ensemble of dimensionless RBM time histories obtained by carrying out 100 simulation cycles is processed to obtain the ensemble characteristics due to randomness in excitation. A plot of ensemble means of 100 RBM time histories (MRBM time history) is given in Fig. 8. The plot of deterministic RBM time history reported in the literature \(^1\) using strip theory and Laplace method of analysis is also given in the same figure. A plot of ensemble variances of 100 RBM time histories (VRBM time history) is given in Fig. 9. It can be seen from Fig. 8 that the values of MRBM obtained from simulations are reasonably in good agreement with the deterministic values given in literature. This
reaffirms the fact that the values of MRBM are not affected by the randomness in gust excitations and remain the same as those obtained using deterministic analysis. However, VRBM time history is the extra information captured by adopting the proposed hybrid Newmark-MCS approach which could not have been otherwise obtained through deterministic analysis. The values of VRBM reflect the effect of randomness in loads on the mean aeroelastic dynamic response of the structure. The results of such an analysis will be useful to carryout reliability analysis and safety assessment of structures.

Fig. 6 Power spectrum of atmospheric turbulence

Fig. 7 A typical realisation of random gust velocity

Fig. 8 Ensemble mean of root bending moment time histories

Fig. 9 Ensemble variance of root bending moment time histories
6. CONCLUSIONS
An approach based on combining Newmark’s method with MCS for realistic estimation of random dynamic response of aircraft structures subjected to random gust excitations is presented. The proposed method is demonstrated by considering the case of an aircraft, which is free in vertical translation with wing being elastic in bending and restrained against twisting. The results of such an analysis will be useful to carryout reliability analysis and safety assessment of structures.

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