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Modelling and parameter estimation for fly-by-wire aircraft/control systems

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Abstract. This paper covers in detail the issues related to parameter estimation of open loop dynamics of fly-by-wire aircraft/control systems from closed loop data. System identifiability aspects in the closed loop and the effect of various feedback types on the parameterisation of the system matrices are reviewed. The methods commonly employed for the detection of collinearity in the data are discussed. A brief discussion of the common methods used for analysis of unstable/augmented aircraft are given. Also, controller information based identification method (CIBIM), which utilises knowledge of the controller in the analysis, is presented. The discussion is followed by numerical results of application of the techniques to simulated data.

Keywords. Fly-by-wire aircraft; control systems; open loop dynamics; parameter estimation; filtering; regression.

1. Introduction

Modelling and identification play very significant roles in present day analysis of complex dynamical systems. Mathematical modelling of any system is necessary to understand its static/dynamical behaviour. The structure of the mathematical model involves parameters characterising the system and these are estimated using estimation techniques. Mathematical modelling of aerospace vehicles is very important since many applications require such information in the form of aerodynamic derivatives that appear in the mathematical model, and which are required for the following reasons (Maine & Iliff 1986).

(i) For explaining aerodynamic stability and control behaviour of the vehicle, thereby describing its static/dynamic behaviour;
(ii) for design of flight control systems, and
(iii) for high fidelity simulators.

Earlier attempts on parameter estimation of linear/nonlinear aerospace vehicles are well reported in the open literature (Jategaonkar & Plaetschke 1983). For stable aircraft applications, most of these methods (Girija & Jategaonkar 1991) work well. Some methods are only suitable for identification of transfer functions of a plant operating in a closed loop control system. In many situations, estimation of parameters of the state space models is required. Determination of aerodynamic derivatives of an aircraft with a fly-by-wire control system (FBWCS, unstable aircraft augmented by feedback) poses new challenges to aircraft identification and parameter estimation problem. When identification experiments are performed with an aircraft operating in a closed loop, the feedback introduces correlations among the input and output variables. This data correlation might cause identifiability problems that render estimates of some derivatives inaccurate (Belsely *et al* 1980; Klein 1989). Due to feedback action, constantly trying to generate controlled responses, the measured responses may not display the modes of the vehicle.

For many applications in the field of flight mechanics, the prediction and determination of the parameters of mathematical models of unstable aircraft is a primary requirement. However, the determination of parameters of open loop plant from the closed loop data presents a variety of problems even when the basic plant is stable. The problem complexity increases when the basic plant is unstable due to the fact that the integration of the state model leads to numerical divergence. The problem is further compounded, if data are noisy, which is the usual case. There are mainly three approaches to the problem of parameter estimation of unstable/augmented systems.

(1) Open loop data can be used directly, ignoring the effect of feedback and, if the feedback loop is tight, this method may give estimates with large uncertainty due to data collinearity (Klein 1989).

(2) An 'optimal input signal' could be designed/generated taking into account the presence of feedback and then the closed loop data could be analysed (Mereau & Abu El Ata-Doss 1985).

(3) Input/output data of the closed loop system with the complete model of the entire system can be used. This involves modelling of control system blocks and nonlinearities. This approach is complicated as it involves higher order system models to be used in the estimation procedure (Koehler & Wilhelm 1979).

In this paper, system identifiability issues in closed loop and the effect of various feedback types on the parameterisation of the system matrices are reviewed. The methods commonly employed for the detection of collinearity in the data are discussed. A brief discussion of the common methods, namely (i) filter error method (FEM), (ii) *ad hoc* stabilisation in output error method, (iii) mixed estimation, and (iv) principal component regression methods, used for analysis of unstable/augmented aircraft is presented. Numerical results of mixed estimation (Girija & Raol 1995a) applied to simulated data and unit uppertriangular diagonal (UD) factorisation-based Kalman filtering applied to unstable systems (Girija & Raol 1993) are presented.

Also, in situations where the control system details are known, they can be utilised in the aggregate model of the system leading to the controller information based identification method (CIBIM) (Girija & Raol 1995b). One of the variants of this method is to estimate equivalent derivatives of the entire system and use the knowledge of the controller to retrieve the open loop dynamics. The results of this method for an unstable aircraft with a simple feedback loop are presented in this paper.

2. Identifiability issues for closed loop systems

The general block diagram of a system operating in closed loop configuration is shown in figure 1. The input (at J), the error signal input (at K) to the plant and the output (at L) are generally measured. The parameters of the plant are to be estimated from these measurements. Two procedures are possible by which the parameters can be estimated from the measured data: (1) Direct identification – a chosen identification method is applied to the data between points K and L, ignoring the presence of the feedback; (2) Indirect identification – the data between points J and L can be used with a chosen identification method to generate equivalent derivatives. Here the closed loop system is regarded as a whole and the parameters are estimated. Using the knowledge of the feedback gain/controllers, the parameters of the plant can be retrieved from the equivalent model. Alternatively, the known models of control systems can be used along with the plant model and unknown parameters of the plant can be determined.

When the direct identification method is used to estimate the parameters of the aircraft operating in closed loop, the feedback introduces correlations among the input and output variables. Noise correlations are also present. This data/noise correlation causes identifiability problems which render estimates of some derivatives inaccurate (Koehler & Wilhelm 1979; Klein 1989). The correlations between the aircraft states and control inputs lead to high correlations between the corresponding stability and control derivatives such that not all the derivatives can be estimated independently (Maine & Murray 1988). Hence, only a degenerate model is identifiable by fixing some of the derivatives at their predicted values, which might result in incorrect estimates in case of wrong predictions.

The control system blocks, namely the feedforward and feedback filters, augment the unstable plant. The effect of feedback on the parameters and structure of the mathematical model is shown in table 1 with the basic plant description given as (Koehler & Wilhelm 1979):

$$\dot{x} = Ax + Bu, \qquad (1)$$

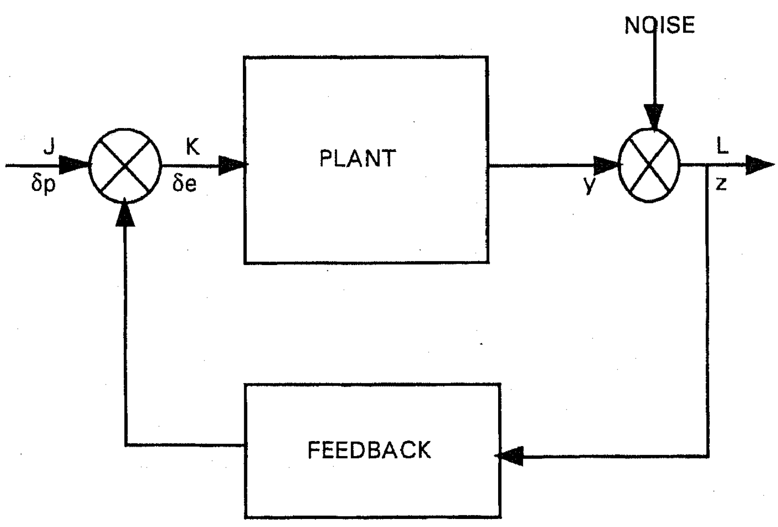

Figure 1. General block diagram of augmented system.

Table 1. Effect of feedback on the parameters and structure of the math model.

Control system type	Input description	Modified sytem	Remarks (changes in)
Constant feedback	$u = Cx + Du_s$	$\dot{x} = (A + BC)x + BDu_s$	Coefficients in the column of feedback
Differential feedback	$u = Cx + E\dot{x} + Du_s$	$\dot{x} = (1 + BE)^{-1}$ $\{(A + BC)x + BDu_s\}$	Almost all coefficients
Integrating feedback	$\dot{u} + Fu = Cx + Du_s$	$\begin{bmatrix} \dot{x} \\ \dot{u} \end{bmatrix} = \begin{bmatrix} A & B \\ C & F \end{bmatrix} \begin{bmatrix} x \\ u \end{bmatrix} + \begin{bmatrix} 0 \\ D \end{bmatrix} u_s$	Structure itself

where x is the $(n \times 1)$ state vector. In table 1, u_s represents the input at point J (figure 1), A is the system matrix, B the control matrix, C the feedback matrix, D the feedforward matrix, and E and F the matrices associated with differential feedback and integrating feedback respectively. From table 1 it is clear that the control system with constant feedback does not affect the structure of the system, but affects only the estimates of the elements of the system matrix A. The modified equations represent a system having a different state matrix. With differential feedback, although the basic structure is the same, all the coefficients are affected even if only one signal is fedback. When the feedback control system has integrators in the feedback loops, the entire structure is changed and the number of poles increases with the number of equations and for very highly augmented systems the order can be very high.

Equation (1) is modified to include the noise:

$$\dot{x} = Ax + Bu + w. \tag{2}$$

If we assume that the feedback is of proportional type (or if the control system dynamics are only weakly excited during the measurement period), the signal u can be represented by

$$u = Cx + Du_s. \tag{3}$$

It is required to estimate the parameters of A, B, C and D. Multiplying (3) by an arbitrary matrix Λ and adding to (2), we get

$$\dot{x} = (A + \Lambda C)x + (B - \Lambda)u + w + \Lambda Du_s. \tag{4}$$

The parameter estimation method treats $(w + \Lambda Du_s)$ as noise and determines the coefficients by minimising the effect of this noise. If the input u_s large, the elements of Λ become insignificant and hence may be neglected. In such a case, (2) and (4) are identical and consequently the feedback has very little influence on the estimated results. However, the large u_s might take the system into nonlinear regions of its behaviour. If the input u_s is small or of short duration, Λ influences the coefficients matrices of x and u. The results of identification will be $(A + \Lambda C)$ and $(B - \Lambda)$ instead of A and B.

When the aircraft responses are correlated due to augmentation system, we have

$$x = Gx, \quad G \neq I, \tag{5}$$

with G as the matrix whose elements could be the feedback gains. Inserting (5) into (2) we get,

$$\dot{x} = [A + \Lambda(G - I)]x + Bu + w. \tag{6}$$

Even here it is difficult to determine the elements of A from output responses, due to the fact that Λ is an arbitrary matrix.

Thus it is clear that any control augmentation causes 'near linear' relationships among variables which are used in the estimation algorithm. It is this collinearity which could affect the accuracy of the estimates. Hence it is required to detect this collinearity in the data and use an appropriate estimation procedure.

3. Collinearity and its detection

An aircraft aerodynamic model can be written in general form as (Klein 1989),

$$y = \theta_0 + \theta_1 x_1 + \cdots + \theta_n x_n, \tag{7}$$

where x_j, $j = 1, 2, \ldots, n$ are the regressors (state and input variables or their combinations), y is a dependent variable (the aerodynamic force or moment coefficient) and $\theta_0 \cdots \theta_n$ are the unknown aerodynamic coefficients. When measured y and x are used, the regression equation takes the form,

$$Y = X\theta + \varepsilon, \tag{8}$$

where Y is $(N \times 1)$ measurement vector, X is the $(N, n+1)$ matrix of regressors and ones, (the ones to account for the constant term in any regression equation), and θ, the $(n+1, 1)$ unknown parameter vector.

The least squares estimates of the parameters θ can be obtained as:

$$\hat{\theta}_{LS} = (X'X)^{-1}X'Y. \tag{9}$$

Regressors X are generally handled by centreing and scaling them to unit length. If X_j^* denotes the columns of the normalised matrix, collinearity means that

$$\sum_{j=1}^{n} k_j X_j^* = 0, \tag{10}$$

for a set of constants k_j not all equal to zero. This collinearity causes computational problems because of ill-conditioning of the matrix and hence inaccurate estimates. For the detection of collinearity in the data X, the methods generally adopted are discussed next.

3.1 *Correlation matrix of regressors*

High correlation coefficient between two regressors can point to a possible collinearity problem. However, if there are several co-existing near-dependencies among regressors, the correlation matrix may be unable to detect collinearity (Belsely *et al* 1980).

3.2 *Eigensystem analysis and singular value decomposition*

For eigensystem analysis (Belsely *et al* 1980), the matrix $X'X$ is decomposed as

$$X'X = M\Delta M'. \tag{11}$$

Here, $'$ denotes transpose of the matrix/vector, Δ is $(n \times n)$ diagonal matrix with its elements as the eigenvalues λ_j of $X'X$ and M is an $(n \times n)$ orthogonal matrix with its columns as the eigenvectors of $X'X$. Eigenvalues close to zero indicate nearly linear dependency in the data. Since it is difficult to specify how small the eigenvalue should be, condition number, which is defined as,

$$K_j = |\lambda_{\max}|/|\lambda_j|, \tag{12}$$

is used as a measure of collinearity. Condition number exceeding 1000 is an indication of severe collinearity.

Because of its better numerical properties, singular value decomposition of matrix X is used to detect collinearity. Here the matrix X is decomposed as

$$X = ZSV', \tag{13}$$

where Z is an $(N \times n)$ matrix and $Z'Z = V'V = I$.

Here, S is an $(n \times n)$ diagonal semi-positive definite matrix with its elements as the singular values of X. Condition index which is defined as

$$\eta_j = \mu_{\max}/\mu_j, \tag{14}$$

is used as a measure of collinearity; values between 5 and 10 indicate mild collinearity and those between 30 and 100 indicate strong collinearity.

3.3 *Parameter variance decomposition*

In this method, the variance of each parameter is decomposed into a sum of components, each corresponding to one and only one of the singular values. The covariance matrix of the parameter estimates θ is given by

$$\mathrm{Cov}(\hat{\theta}) = \sigma_r^2 (X'X)^{-1} = \sigma_r^2 M \lambda^{-1} M', \tag{15}$$

where σ_r^2 is the residual variance.

The variance of each parameter is equal to

$$\sigma_{\theta_j}^2 = \sigma_r^2 \sum_{k=1}^{n} \frac{t_{jk}^2}{\lambda_j} = \sigma_r^2 \sum_{k=1}^{n} \frac{t_{jk}^2}{\mu_j^2}, \tag{16}$$

where t_{jk} are the elements of the eigenvector t_j associated with λ_j Equation (16) decomposes the variance of each parameter into a sum of components, each corresponding to one and only one of the n singular values. Since μ_j appears in the denominator, one or more small singular values can increase the variance of θ_j. Hence an unusually high property of the variance of two or more coefficients for the same small singular value can provide evidence that the corresponding near-dependency causes problems.

Defining

$$\phi_{jk} = t_{jk}^2/\mu_j^2 \quad \text{and} \quad \phi_j = \sum_{k=1}^{n} \phi_{jk}, \tag{17}$$

one gets the j, k variance-decomposition proportion as the proportion of the variance of the jth regression coefficient associated with the kth component of its decomposition in (17) and is given by

$$\Pi_{kj} = \phi_{jk}/\phi_j; \quad j, k = 1, 2, \ldots, n. \tag{18}$$

Since two or more regressors are required to create near-dependency, two or more variances will be adversely affected by high variance-decomposition proportions associated with a singular value. Variance proportions greater than 0.5 are recommended as guidelines for possible collinearity problems (Belsely *et al* 1980).

4. Methods for parameter estimation of unstable/augmented systems

The output error method (OEM) is most widely used for the determination of stability and control derivatives of aircraft. OEM requires numerical integration of unstable equations of motion of the plant and this in turn causes numerical divergence. Hence, although in principle, OEM is applicable to unstable system analysis, special care has to be taken to avoid the problem of numerical divergence of the algorithm. One such method it to employ *ad hoc* stabilisation in OEM (Maine & Murray 1988). An alternative is to use the filter error method (Jategaonkar & Plaetschke 1989). OEM is a special case of the filter error method described below.

4.1 *Filter error method*

The filter error method (FEM) is practical for linear systems which have the general form:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + w; \quad x(0) = x_0; \quad w \sim N(0, FF'), \\ z(t_i) &= Hx(t) + v, \quad i = 1, 2, \ldots, N; \quad v = N(0, GG'). \end{aligned} \tag{19}$$

The maximum likelihood estimates of $\theta = \{A, B, H, G\}$ for this system are obtained by minimising the cost function:

$$J(\Theta) = \sum_{i=1}^{N} [z(t_i) - \hat{z}(t)]^T R^{-1} [z(t_i) - \hat{z}(t_i)], \tag{20}$$

where R is the innovation covariance matrix and $\hat{z}(ti)$ is predicted using the (steady state) Kalman filter (Jategaonkar & Plaetschke 1989):

$$\hat{x}(t_0) = \hat{x}_0, \tag{21}$$

$$\tilde{x}(t_i) = \phi\tilde{x}(t_{i-1}) + B_d u(t_{i-1}), \tag{22}$$

$$\tilde{z}(t_i) = H\tilde{x}(t_i), \tag{23}$$

$$\tilde{x}(t_i) = \tilde{x}(t_i) + K[z(t_i + K[z(t_i) - \tilde{z}(t_i)], \tag{24}$$

where θ is the transition matrix and B_d the discrete control matrix, '$\sim$' represents the predicted variables and '$\wedge$' the estimated variables.

The steady state Kalman gain is given by,

$$K = PH'(HPH' + GG')^{-1}, \tag{25}$$

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where θ is the transition matrix and B_d the discrete control matrix, '$\sim$' represents the predicted variables and '$\wedge$' the estimated variables.

The steady state Kalman gain is given by,

$$K = PH'(HPH' + GG')^{-1}, \tag{25}$$

where P is the steady-state covariance of the predicted state, given as the limit of the Ricatti equation (Maine & Murray 1989):

$$P_{i+1} = \phi[P_i - P_i H'(H P_i H' + GG')^{-1} H P_i]\phi' + FF'. \tag{26}$$

The above equation converges to a unique solution independent of the initial covariance and the filter is stable if all the unstable modes of the system are observable and controllable. Filter error method reduces to OEM for stable systems with no process noise. For unstable systems with $FF' = 0$, the solution is not unique. For nonsingular FF', steadystate solutions are unique and smoothly approach the nonzero steady state solution. OEM works with $F = 0$, if all the aircraft modes are observable and there are no neutrally stable modes. The limitation of FEM is that it is only practical for linear systems and involves considerable programming effort.

4.2 *Ad hoc stabilisation method in OEM*

The stabilisation of the filter in FEM is due to the feedback term which is proportional to the error between the measured and the estimated states (also called fit error). The filter feedback helps stabilise the analysis algorithm and does not bear any relation to the control system feedbacks that stabilise the aircraft. Stabilising feedback of fit errors can alternatively be implemented in the mathematical models used in identification software. These *ad hoc* feedback terms could include nonlinearities and other practical engineering judgement to improve the results.

The *ad hoc* method has a form somewhat similar to FEM and gives similar results but is applicable to many practical situations if proper care is taken to obtain appropriate feedback gains. For the linear system described by (13), the predicted state is:

$$\tilde{x}(t_i) = \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}), \tag{27}$$

$$\tilde{z}(t_i) = H\tilde{x}(t_i). \tag{28}$$

By introducing *ad hoc* stabilisation by software feedback of a state in the mathematical model, we have the input to the model:

$$u(t_{i-1}) = u(t_{i-1}) + K_{SF}\tilde{x}(t_{i-1}), \tag{29}$$

where K_{SF} is the software gain. Hence,

$$\begin{aligned}\tilde{x}(t_i) &= \phi\tilde{x}(t_{i-1}) + B_d u(t_{i-1}) + B_d K_{SF}\tilde{x}(t_{i-1}) \\ &= (\phi + B_d K_{SF})\tilde{x}(t_{i-1}) + B_d u(t_{i-1}).\end{aligned} \tag{30}$$

If K_{SF} is properly chosen, the system $(\phi + B_d K_{SF})$ is made locally stable in the mathematical model of the original plant ϕ which may be unstable.

In FEM, substituting (22) in (24), we have,

$$\hat{x}(t_i) = \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}) + KH[x(t_i) - \tilde{x}(t_i)]. \tag{31}$$

Simplifying after substituting for $\tilde{x}(t_i)$ we have (ignoring the noise terms):

$$\begin{aligned}\hat{x}(t_i) &= \phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}) + KH\phi[x(t_{i-1}) - \hat{x}(t_{i-1})] \\ &= (I - KH)\phi\hat{x}(t_{i-1}) + KH\phi\hat{x}(t_{i-1}) + B_d u(t_{i-1}).\end{aligned} \tag{32}$$

Equations (30) and (32) have somewhat similar form. Hence it follows that in the adhoc stabilisation method, (30), the software feedback produces a stabilising effect similar to that produced by the feedback of the fit error in the Kalman filter in FEM.

4.3 *Mixed estimation*

The method of mixed estimation (Klein 1989) augments the measured data by *a priori* information on the parameters directly. Assuming that $p \leq n$ (n is the total number of parameters to be estimated) prior information on the elements of θ are available, the *a priori* information equation (PIE) can be formulated as,

$$a = C\theta + \zeta, \tag{33}$$

where a is a p-vector of known *a priori* values, C is a matrix of each $p \leq n$ which includes known constants and ζ is a random vector with $E(\zeta) = 0$, $E(\zeta\varepsilon') = 0$, $E\{\zeta\zeta\} = \sigma^2 W$, where W is a known weighting matrix. Here E stands for mathematical expectation.

Combining (8) and (33), the mixed model is given by

$$\begin{bmatrix} Y \\ a \end{bmatrix} = \begin{bmatrix} X \\ C \end{bmatrix} \theta + \begin{bmatrix} \varepsilon \\ \zeta \end{bmatrix}. \tag{34}$$

Applying the least squares method to (34), the mixed estimates (ME) are obtained as

$$\hat{\theta}_{\mathrm{ME}} = (X'X + C'W^{-1}C)^{-1}(X'Y + C'W^{-1}a), \tag{35}$$

with the covariance matrix given by

$$\mathrm{Cov}(\hat{\theta}_{\mathrm{ME}}) = \sigma^2[X'X + C'W^{-1}C]^{-1}. \tag{36}$$

In practical applications, PIE may not be known exactly and hence the resulting estimator is biased. Generally the W matrix is diagonal with the elements representing uncertainty of *a priori* values.

4.4 *Principal components regression method*

Here the original regressors x_j are transformed into the space of orthogonal regressors z_j (Klein 1989). The regression model after transformation becomes,

$$Y = Z\gamma + \varepsilon, \tag{37}$$

where $Z = XT$ and $\theta = T\gamma$, T is the eigenvector matrix of the $X'X$ matrix and γ is the set of parameters associated with the orthogonal regressors. Columns of Z matrix are called 'Principal Components'. The estimates are obtained by

$$\hat{\gamma} = (Z'Z)^{-1}Z'Y = \Lambda^{-1}Z'Y, \tag{38}$$

and the covariance matrix of $\hat{\gamma}$ is

$$\mathrm{Cov}(\hat{\gamma}) = \sigma^2\Lambda^{-1}. \tag{39}$$

To get the principal component estimates, the regressors are arranged in order of decreasing eigenvalues λ_j, the principal components associated with small eigenvalues are deleted and the LS is applied to the remaining components. The principal component estimator has the form

$$\hat{\theta}_{PC} = \sum_{j=1}^{n-r} (1/\lambda_j)' t_j' X' Y t_j. \tag{40}$$

When all data are ill conditioned, the principal component regression yields better estimates than the conventional LS techniques.

In this paper, the mixed estimation technique is used for parameter estimation along with the collinearity diagnostics. This is quite suitable for aircraft applications, since the *a priori* information required for this method is available from wind tunnel/analytical predictions or from previous flight test experiments/analysis.

4.5 *Numerical validation of ME method*

All the collinearity diagnostics described in § 3 are implemented in PC MATLAB. The least squares mixed estimation (LSME) method is also implemented in PC MATLAB. OEM program, based on the maximum likelihood method (Jategaonkar & Plaetschke 1983), is used to analyse the simulated data to generate results for comparison purposes for augmented dynamical systems.

Figure 2 shows the block diagram of a typical augmented longitudinal dynamics of an aircraft. Block 1 in the figure is for pilot command shaping, block 2 is actuator dynamics with delay, block 3 represents the aircraft dynamics and blocks 4–7 are feedback filters.

Example 1. Second order short period dynamics of an aircraft having the following state and observation equations are considered for generating simulated data.

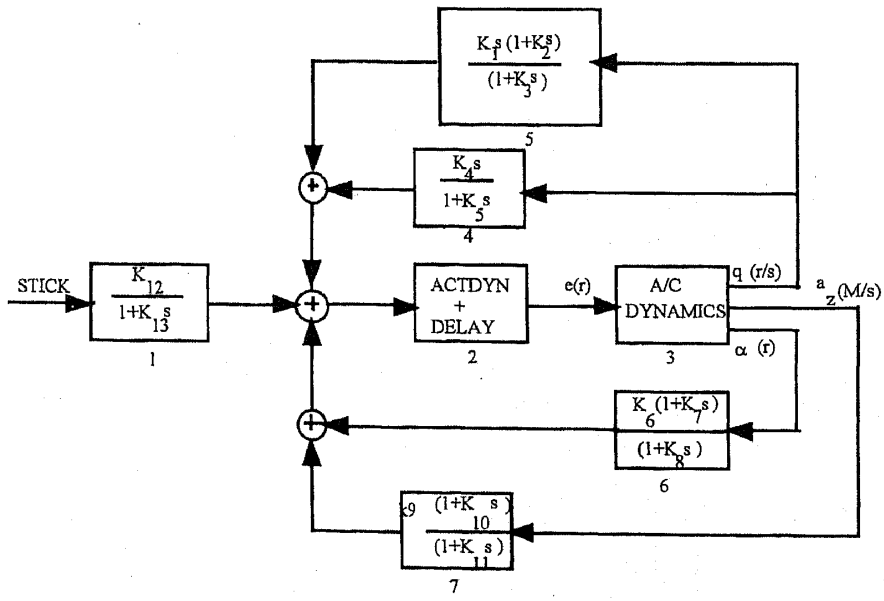

Figure 2. Block diagram of simulated closed loop system.

State equations:

$$\dot{\alpha} = Z_\alpha \alpha + q + Z_{\delta_e}\delta_e, \tag{41}$$

$$\dot{q} = M_\alpha \alpha + M_q q + M_{\delta_e}\delta_e. \tag{42}$$

Observation equations:

$$\alpha_m = \alpha,$$

$$q_m = q, \tag{43}$$

where α is the angle of attack, q is the pitch rate and δ_e is the elevator (control surface) deflection. Z_α, Z_{δ_e}, M_α, M_q, M_{δ_e} are the aerodynamic derivatives to be estimated. For generating simulated data, only the α signal is fed back to the control surface through a gain K_α so that we have the feedback signal given by

$$\delta_e = K_\alpha \alpha + \delta_p. \tag{44}$$

By varying the gain K_α different sets of data are generated for analysis. Typical time histories of α, q, δ_e, when the feedback gain $= 1$, and δ_p (pilot input) is a doublet signal, are shown in figure 3 .Sampling period $= 0.1$ s and the data record length of 10 s is used in the analysis.

The closed loop eigenvalues of the system are shown in table 2 for a range of the feedback gain ($K_\alpha = 0$ to 1). The parameters estimated using OEM and LS from the open loop ($K_\alpha = 0$) and SNR $= \infty$ and SNR $= 10$ are given in table 3. The parameter estimation error norms (PEEN) are defined as,

$$\text{PEEN}(\%) = 100^*\text{norm}(b_t - b_e, i)/\text{norm}(b_t, i), \tag{45}$$

where b_t = vector of true parameters, b_e = vector of estimated parameters, and if $i = 1$, it is L1 norm and for I = 2 it is L2 norm.

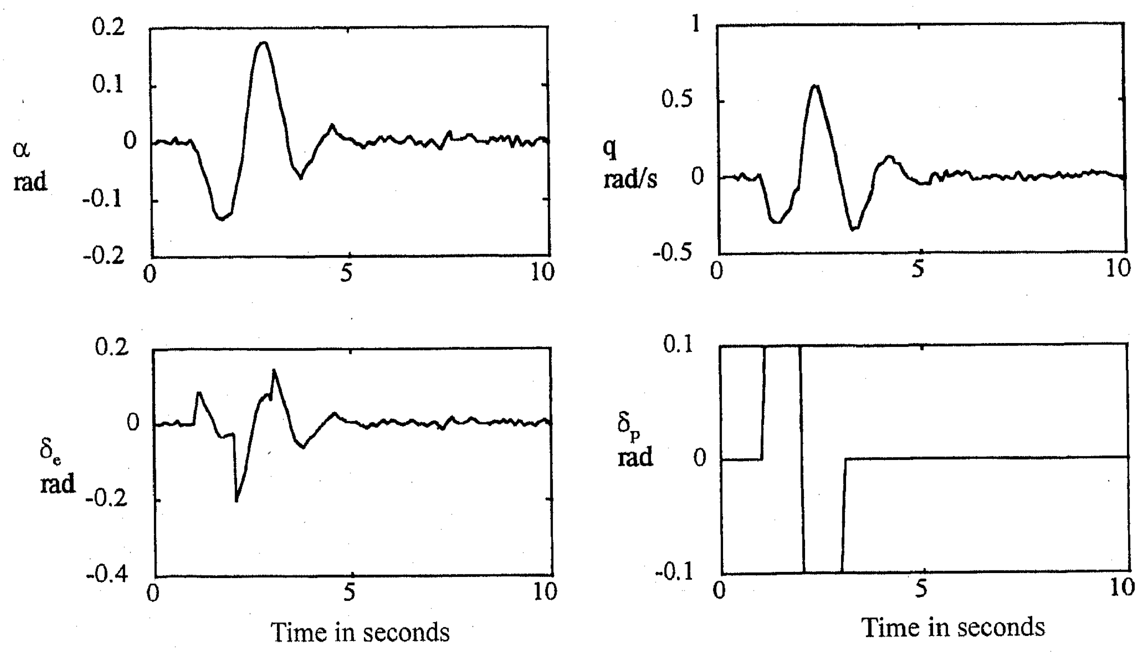

Figure 3. Time history plots (example 1, second-order short period dynamics).

Table 2. Closed loop eigenvalues of the second order system (example 1).

Gain K_α	Eigenvalues	ω_n rad/s	ζ
0.0	$-0.2881, -1.7443$	–	–
0.03	$-0.7188, -1.3265$	–	–
0.07	$-1.0318 \pm j0.7256$	1.2164	0.8180
0.15	$-1.0485 \pm j1.2882$	1.6610	0.6312
0.3	$-1.0809 \pm j1.9609$	2.2390	0.4827
1.0	$-1.2315 \pm j3.7435$	3.9409	0.3125

The estimated derivatives using OEM and LS for the case when SNR $= \infty$ are close to the true values. As the SNR decreases, the OEM estimates show some deviations and the standard deviation of the parameter estimates increases. However, the LS estimates show a much larger deviation from the true values. This is because the LS method generates biased estimates when the regressors are noisy. The L1 and L2 norms also indicate an increase in error as the signal-to-noise ratio decreases.

Table 4 gives the estimates using OEM and LS (for $K_\alpha = 1$). For SNR $= \infty$, OEM estimates are close to true values whereas the LS estimates show larger deviations. As the SNR decreases, standard deviations of the parameters increase for OEM and LS methods, and the parameter estimates deviate from the true values. The L1 and L2 norms also increase as the SNR decreases. This behaviour can be attributed to the correlations of signals (and noise) due to feedback.

Since the data are generated using closed loop plant, the collinearity diagnostics described in § 3 are computed to assess the extent of collinearity in the data. The correlation matrix and variance proportions for one typical case of SNR $= 100$ are given in tables 5 and 6. The correlation matrix and variance proportions are computed assuming there is a constant term in the regression equation in addition to the states α, q, δ_e. In table 6, the condition numbers are also indicated. In figure 4 these values are plotted. The

Table 3. Parameter estimates (example 1, $K_\alpha = 0$).

		SNR			
		OEM		LS	
Parameter	True value	∞	10	∞	10
Z_α	−0.9624	−0.9602 (0.0004)	−0.8732 (0.07)	−0.9721 (0.037)	−0.4392 (0.45)
M_α	0.5273	0.5298 (0.0007)	0.5819 (0.11)	0.1947 (0.11)	−0.1200 (0.75)
M_q	−1.0698	−1.0743 (0.0005)	−1.2214 (0.10)	−0.8601 (0.07)	−1.7730 (0.49)
Z_{δ_e}	−0.4315	−0.4361 (0.0027)	−0.6084 (0.24)	−0.3844 (0.01)	−2.2503 (1.54)
M_{δ_e}	−14.5747	−14.5951 (0.003)	−15.5875 (0.73)	−13.0695 (0.32)	−9.5054 (2.56)
L1%	–	0.1947	8.4535	11.9800	55.8076
L2%	–	0.1477	7.1241	10.6200	38.80