Multiple Pole Rational-Function Approximations for Unsteady Aerodynamics

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Nomenclature

\[ [A] = \text{coefficient matrices} \]
\[ b = \text{reference length} \]
\[ b_r = \text{poles (lag-parameters)} \]
\[ [Q] = \text{unsteady aerodynamic transfer-function matrix} \]
\[ Q_s = \text{element (i, j) of [Q]} \]
\[ s = \text{Laplace variable} \]
\[ U = \text{freestream velocity} \]

Introduction

For a general aeroviscoelastic analysis, the equations of motion are desired in a linear, time-invariant, state-space form. This necessitates the representation of the unsteady aerodynamic transfer function matrix, for a general motion in the Laplace domain, by a rational-function approximation (RFA) for each term of the matrix. Since the unsteady aerodynamic transfer-function matrix \([Q(s)]\) is analytic for a causal, stable, and linear system, it can be directly deduced from the frequency domain data through a process of analytic continuation, which involves a least squares curve-fit.

Several approaches have been used to determine the poles (lag-parameters) of \([Q(s)]\) by a nonlinear optimization process. Dunn,1 Karpe1, and Peterson and Crawley5 used gradient-based optimization schemes, whereas, Refs. 4–6, and 9 employed Simplex nongradient techniques. Peterson and Crawley1 observed the phenomenon of repeated poles approximating for the Thed-ersen function. However, the repeated lag-states mistakenly indicated that the same accuracy can be achieved by reducing the number of lag-states. Eversman and Tewari1 encountered the repeated values of lag-parameters frequently in a nongradient-optimized RFA and correctly identified the phenomenon to indicate the need for a new multiple-pole approximation in the Laplace domain. Reference 5 showed that while the conventional approximation of simple poles produces an ill-conditioned eigenvalue problem for the state-space model when the poles are close to one another, the new multiple-pole RFA accounts for such cases consistently. Additionally, the use of multiple poles resulted in a large reduction in the optimization and, while preserving the fit accuracy and the total number of aerodynamic states when compared to the conventional approximation. Eversman and Tewari1 also presented improved and consistent RFA for the Thedersen function by using the multiple-pole approximation. Tewari1 in a Ph.D. dissertation showed that the multiple-pole RFA is needed not only in the subsonic regime, but also for supersonic speeds. References 5 and 9 arrived at the multiple-pole RFA through numerical considerations. The present work examines the multiple-pole RFA from a mathematical standpoint and validates its need by concluding that multiple-pole RFAs is dictated in the function space by the constrained optimization theory.

Numerical Need for Multiple-Pole RFA

A simple-pole, least-squares RFA for the unsteady aero-
dynamic transfer-function matrix can be expressed as

\[
[Q(s)] = [A_0] + [A_1]s^{-1} + \ldots
\]

(1)

Reference 5 showed that the optimized values of two or more lag parameters \(b_r\) frequently tend to be close to one another for a subsonic numerical test case. It was also shown that when repeated poles occur, numerical considerations point toward the need for a multiple-pole RFA, given by

\[
[Q(s)] = [A_0] + [A_1]s^{-1} + \ldots + [A_{N_p}]s^{-N_p} + \ldots
\]

(2)

where \(N_p\) is the total number of poles, \((N_1 - N_p)\) the number of poles repeated twice or more times, etc. Such RFA avoid the ill-conditioned eigenvalue problem produced by having repeated poles in Eq. (1).

While Ref. 5 studied the RFA for the subsonic case, it was subsequently discovered1' that repeated poles are equally prevalent in the supersonic regime, and that a multiple-pole RFA, given by Eq. (2), produces a consistent and efficient approximation for supersonic speeds. In the test-case considered, the same planform geometry was used as in Ref. 5, but the wing was stiffened in order to have the flutter-speed in the supersonic regime. The supersonic "doublet-point" method was used to generate the frequency domain data at the following set of reduced frequencies:

\[
0.0, 0.05, 0.1, 0.125, 0.175
\]
\[
0.2, 0.3, 0.5, 0.7, 0.9, 1.2
\]

Only the first six structural modes were retained. Table 1 presents the optimum pole values for supersonic Mach numbers. As in Ref. 5 for subsonic Mach numbers, it is noted...
Table 1: Optimum lag-parameter values for supersonic Mach numbers

<table>
<thead>
<tr>
<th></th>
<th>b₁</th>
<th>b₂</th>
<th>b₃</th>
<th>b₄</th>
<th>b₅</th>
<th>b₆</th>
<th>b₇</th>
<th>b₈</th>
</tr>
</thead>
<tbody>
<tr>
<td>M</td>
<td>1.05</td>
<td>0.3591</td>
<td>0.85623</td>
<td>0.85627</td>
<td>0.45676</td>
<td>0.45202</td>
<td>0.46029</td>
<td>0.45670</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>0.3487</td>
<td>0.90694</td>
<td>0.95183</td>
<td>0.2919</td>
<td>0.29429</td>
<td>0.21630</td>
<td>0.22926</td>
</tr>
<tr>
<td></td>
<td>1.2</td>
<td>0.61702</td>
<td>0.06371</td>
<td>0.87254</td>
<td>0.10342</td>
<td>0.42278</td>
<td>0.11742</td>
<td>0.11751</td>
</tr>
<tr>
<td></td>
<td>1.3</td>
<td>0.33952</td>
<td>0.24700</td>
<td>1.23706</td>
<td>0.17274</td>
<td>0.17274</td>
<td>0.55536</td>
<td>0.34634</td>
</tr>
<tr>
<td></td>
<td>1.4</td>
<td>0.34735</td>
<td>0.34871</td>
<td>1.80238</td>
<td>0.83568</td>
<td>0.77938</td>
<td>1.82796</td>
<td>0.16268</td>
</tr>
<tr>
<td></td>
<td>1.5</td>
<td>0.45369</td>
<td>0.96795</td>
<td>0.96795</td>
<td>0.00108</td>
<td>0.28711</td>
<td>0.10182</td>
<td>0.127046</td>
</tr>
<tr>
<td></td>
<td>1.6</td>
<td>0.39346</td>
<td>0.73422</td>
<td>0.73489</td>
<td>0.17320</td>
<td>0.17401</td>
<td>0.96374</td>
<td>0.90555</td>
</tr>
</tbody>
</table>

Table 2: New lag-parameters for M = 1.85

<table>
<thead>
<tr>
<th>Number of lag-parameters,</th>
<th>Eq. (1)</th>
<th>Lag-parameters,</th>
<th>Eq. (2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.85029</td>
<td>0.85029 (Double-pole)</td>
<td>(Fit-error = 16.901)</td>
</tr>
<tr>
<td></td>
<td>0.50756</td>
<td>0.50756 (Triple-pole)</td>
<td>(Fit-error = 5.498)</td>
</tr>
<tr>
<td></td>
<td>0.451749</td>
<td>0.451749</td>
<td>(Fit-error = 5.212)</td>
</tr>
<tr>
<td></td>
<td>0.44089</td>
<td>0.439976 (Triple-pole)</td>
<td>(Fit-error = 4.344)</td>
</tr>
<tr>
<td></td>
<td>0.45670</td>
<td>0.000106 (Simple-pole)</td>
<td>(Fit-error = 4.365)</td>
</tr>
<tr>
<td></td>
<td>0.00022</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Analytical Look at the Need for Multiple Poles

It is not surprising from the mathematical viewpoint that the need for a multiple-pole RFA should frequently arise. Let Ω be the set of all rational-functions with a fixed order of poles. The rational-function with multiple-poles occur on the boundary of Ω (Fig. 1). The nonlinear optimization problem for the determination of optimum poles can be considered as the minimization of the least-squares fit-error of the RFA with the frequency-domain data, subject to the constraint Q ∈ Ω. This type of constraint is referred to as a set-constraint. Since the constrained optimization process often yields solutions on the boundary of the feasible set, which is defined as the set of all multiple-pole rational-functions, it follows that the optimal rational-functions would frequently have multiple poles.

Conclusion

The phenomenon of repeated poles (lag-parameters) in a rational-function approximation for the unsteady aerodynamic transfer-function matrix is frequently encountered not only at subsonic but also at supersonic speeds. As with the subsonic case, a multiple-pole approximation accurately, efficiently, and consistently replaces the conventional, simple-pole approximation in the supersonic regime. When considered from the mathematical perspective of constrained, nonlinear function minimization theory, the observed need for multiple-pole RFA is easily explained. The multiple-pole rational-functions form a boundary for the set of all rational-functions (with the same order of poles), which is also the feasible set for the optimization problem in the function space.

References

7. Ueda, T., and Dowell, E. H., “Double-Point Method for Super-
Neutrally Reinforced Holes in Symmetrically Laminated Plates

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Introduction

Since laminated composite plates with openings are widely used structural elements in many engineering applications, the analysis of such structures has been done by numerous researchers. The stress and strain state around the openings have been presented for a variety of problems. A large number of problems have been presented in the monographs by Lekhnitskii and Savin. In the formulation of these problems, the material constitutive law, the geometry, loading, and boundary conditions are assumed to be given and the stress and strain state are computed.

In a particular class of structural optimization, the geometry of the body is obtained as a solution to a particular problem, for known states of stress, strain, boundary condition and appropriate constraints (sometimes called inverse problems for some specific cases). In the latter category, either stress concentrations are reduced (optimized) by determining the shape of the openings (harmonic holes), or stress states in the cut structures with reinforcement are maintained unchanged to that of the uncut structures (neutral holes).

All previous solutions for neutral holes have been obtained for elastic, isotropic sheets under planar states of stress. In a previous paper, the methodology outlined in Ref. 1 was extended to symmetrically laminated composite plates which are under planar loading. In the present work, the case of a symmetrically laminated plate subjected to pure bending moments is considered. It must be noted that the notion of a neutral hole in the context of plate flexure assumes a new meaning. Here, the purpose is to introduce a reinforced cutout into a plate that will maintain the same moment and curvature distributions as of the uncut plate throughout.


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