Flutter of Skew Panels by the Matrix Displacement Approa h

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Introduction

In spite of the large number of published works on panel flutter[1] there appears to be a wide gap in the literature concerning the flutter of skew panels. For instance, recently the flutter[1] behaviour of skew panels in supersonic flow has been presented[2] for a simply-supported boundary condition using double Fourier sine series to represent the deflection surface. Apart from this publication there is practically no literature concerning flutter of skew panels except refs. 3 4 which consider the flutter of skew panels clamped on all the edges. The method used in ref. 3 is the common 4-mode analysis by using the Iguchi functions for representing the deflections and in ref. 4 the same problem is solved by the use of beam characteristic functions. One inherent difficulty in these conven methods, was that no single function could be chos represent the deformation which satisfied various bou conditions, with the result that the entire analysis have to be repeated with different assumed functio accommodating different boundary conditions. Her general method was proposed in ref. 5 for the stu panel flutter problems of arbitrary geometry by the M Displacement Method, which permits application to lems with practically any geometrical boundary cond on any or all the sides.

The present note considers the practical applicati the general method to Ihe problem of supersonic i and vibration of skew panels.

The basic elements considered in this note ar parallelograms (fig. 1) for which the natural stif inertia and aerodynamic influence coefficient

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matrices can be derived as explained in refs. 5 and 6. The assembled matrices, which refer to the complete panel, may then be achieved elegantly from the elemental matrices with the help of Boolean matrix operations. After obtaining the required stiffness, inertia and AIC matrices, the equations of motion under the influence of elastic, inertial and aerodynamic forces can be formed in the form of matrix algebra and solved for the complex eigenvalues and vectors.

Notation

- **a** Boolean matrix describing topological connections of the element
- a length of the element along the x-direction
- **A** assembled aerodynamic damping matrix (symmetric)
- $\mathbf{A}_0$ elemental aerodynamic damping matrix (symmetric)
- b length of the element along the y-direction
- **B** assembled aerodynamic stiffness matrix (skew)
- $\mathbf{B}_0$ elemental aerodynamic stiffness matrix (skew)
- $C_0 = ql^3/2\beta D$
- $C_1 = (M^2-2)\, C_0/\beta^2$
- $C_2 = (\beta/4)\,(\rho_m/\rho_a)\,(h/l)\, C_0$
- D plate rigidity
- h thickness of panel
- k non-dimensional frequency
- $\mathbf{K}_E$ elastic stiffness matrix (symmetric)
- l reference length
- M Mach number
- $\mathcal{M}$ inertia matrix (symmetric)
- p intensity of aerodynamic pressure
- $q = \frac{1}{2}\rho_a U^2$, dynamic pressure
- **q** generalised displacement vector, a column matrix
- $\tilde{\mathbf{q}} = \mathbf{q}e^{\bar{\lambda} t}$
- $Q = 4C_0/\pi^4$, aerodynamic parameter
- U free-stream velocity
- $w = W/l$, non-dimensional displacement
- W vertical displacement
- x, y, z oblique co-ordinates of the element
- X, Y, Z rectangular co-ordinates of the panel
- α damping factor
- $\beta = (M^2-1)^{1/2}$
- Λ angle of skew
- ω circular frequency
- λ, μ non-dimensional lengths of the element
- $\bar{\lambda}$ complex number, $\left(\frac{\alpha l}{U} + i\,\frac{\omega l}{U}\right)$
- $\bar{\mu}$ complex eigenvalue, $(\mu_R + i\mu_I)$
- ξ, η non-dimensional oblique co-ordinates of the element
- ρ_a density of air
- ρ_m density of panel material
- $\boldsymbol{\rho}$ column vector of kinematic modes of an element (12×1)
- $\boldsymbol{\Omega}$ transformation matrix function (1×12)
- *Dot* $(\dot{\ })$ denotes differentiation w.r.t. physical time t
- *subscript* ξ differentiation with reference to ξ
- *superscript* t transposed matrix

Equations of Motion and Its Solution

The dynamic equations, under the influence of stiffness, inertia and aerodynamic forces may be written as

$$[\mathbf{K}_E]\,\tilde{\mathbf{q}} + C_0\,[\mathbf{B}]\,\tilde{\mathbf{q}} + C_1\,[\mathbf{A}]\,\dot{\tilde{\mathbf{q}}} + C_2\,[\mathcal{M}]\,\ddot{\tilde{\mathbf{q}}} = 0. \quad (1)$$

The details regarding the derivation of elastic stiffness

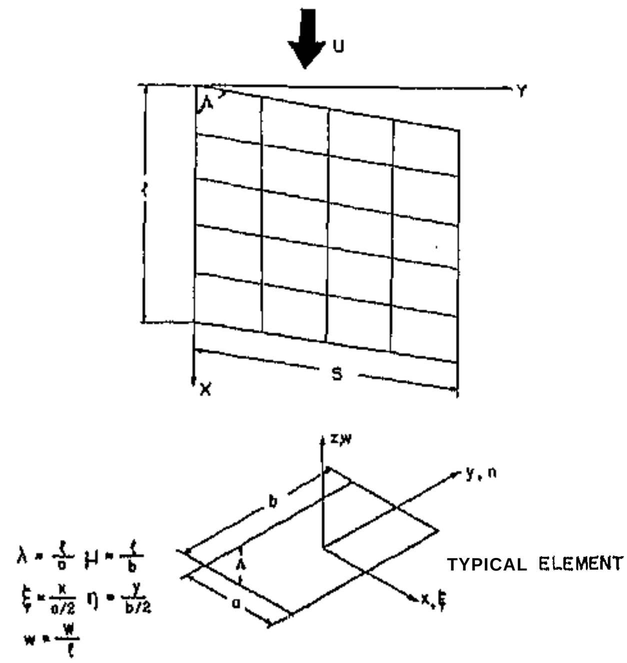

Figure 1. Idealisation of a panel.

$[\mathbf{K}_E]$ and inertia $[\mathcal{M}]$ matrices may be obtained from ref. 6 and that of; AIC matrices [A] and [B] from ref 5. But for the sake of clarity, the derivation of AIC matrices is mentioned briefly here.

The dynamic pressure intensity at any point (x, y) may be represented as

$$p\,(x, y, t) = \frac{\rho_a U^2}{\sqrt{(M^2-1)}}\left[\frac{\partial W}{\partial x} + \frac{M^2-2}{M^2-1}\,\frac{1}{U}\,\frac{\partial W}{\partial t}\right] \quad . \quad (2)$$

Using non-dimensional parameters

$$\xi = \frac{x}{a/2}, \quad \eta = \frac{y}{b/2}, \quad \lambda = \frac{l}{a}, \quad \mu = \frac{l}{b}$$

and $w = W/l$

and representing the non-dimensional deflection w as

$$w = \boldsymbol{\Omega}\boldsymbol{\rho}$$

where $\boldsymbol{\rho}$ stands for the (12×1) column vector of kinematic modes[6] and $\boldsymbol{\Omega}$ a matrix transformation function (*see* Appendix), it is possible from the principle of virtual work, to derive the kinematically consistent aerodynamic forces as

$$\mathbf{p} = \frac{ql^3 \sin\Lambda}{2\beta}\left[\mathbf{B}_0\boldsymbol{\rho} + \frac{M^2-2}{\beta^2}\,\frac{l}{U}\,\mathbf{A}_0\dot{\boldsymbol{\rho}}\right] \quad . \quad (3)$$

where

$$\mathbf{B}_0 = \frac{1}{\lambda\mu}\int_{-1}^{+1}\int_{-1}^{+1} \boldsymbol{\Omega}^t\,\boldsymbol{\Omega}_\xi\, d\xi\, d\eta = \mathbf{a}_0{}^t\overline{\mathbf{B}}_0\mathbf{a}_0 \quad . \quad (4)$$

$$\mathbf{A}_0 = \frac{1}{\lambda\mu}\int_{-1}^{+1}\int_{-1}^{+1} \boldsymbol{\Omega}^t\,\boldsymbol{\Omega}\, d\xi\, d\eta = \mathbf{a}_0{}^t\overline{\mathbf{A}}_0\mathbf{a}_0 \quad . \quad (5)$$

and $\mathbf{a}_0$ is a transformation matrix[6].

The details of matrices $\overline{\mathbf{A}}_0$ and $\overline{\mathbf{B}}_0$ are given in Tables I and II, Having obtained the AIC matrices $\mathbf{B}_0$ and $\mathbf{A}_0$ for discrete elements, it is straightforward to assemble them by means of Boolean operations to obtain the big matrices for the complete panel, i.e.

$$\mathbf{A} = \mathbf{a}^t\mathbf{A}_0\mathbf{a}, \quad \mathbf{B} = \mathbf{a}^t\mathbf{B}_0\mathbf{a} \quad . \quad . \quad (6)$$

TABLE I

MATRIX $\overline{A}_0$—(AIC)

$\widetilde{A}_0 =$

1/120	1/144	0	0	0	0	0	0	0	1/24	0	n
1/144	1/120	0	0	0	0	0	0	0	1/24	0	0
0	0	1/840	0	0	1/560	0	0	0	0	1/240	o
0	0	0	1/840	1/560	0	0	0	0	0	0	-1/240
0	0	0	1/560	17/4200	0	0	0	0	0	0	-1/120
0	0	1/560	0	0	17/4200	0	0	0	0	1/120	0
0	0	0	0	0	0	17/29400	9/19600	1/300	0	0	n
0	0	0	0	0	0	9/19600	17/29400	1/300	0	0	0
0	0	0	0	0	0	1/300	1/300	1/36	0	0	0
1/24	1/24	0	0	0	0	0	0	0	1/4	0	ft
0	0	1/240	0	0	1/120	0	0	0	0	1/48	0
0	0	0	-1/240	-1/120	0	0	0	0	0	0	1/48

$d_1 = [1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1 \;\; 1 \;\; 1/\lambda \;\; 1/\mu]$

$$\overline{A}_0 = \frac{1}{\lambda\mu} \, d_1 \widetilde{A}_0 d_1$$

TABLE II

MATRIX B_0—(AIC)

$\widetilde{B}_0 =$

0	0	1/120	0	0	1/60	0	0	0	0	1/24	0
0	0	0	0	0	1/60	0	0	0	0	1/24	0
−1/120	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	1/420	1/60	0	0	0
0	0	0	0	0	0	17/4200	3/700	1/30	0	0	0
−1/60	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	−17/4200	0	0	0	0	0	0	0
0	0	0	0	−3/700	0	0	0	0	0	0	0
0	0	0	0	−1/30	0	0	0	0	0	0	0
0	0	0	0	0	1/12	0	0	0	0	1/4	0
−1/24	0	0	0	0	0	0	0	0	0	0	0
0	0	t)	0	0	0	0	−1/120	−1/12	0	0	0

$d_1 = [1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1/\lambda \;\; 1/\mu \;\; 1 \;\; 1 \;\; 1/\lambda \;\; 1/\mu]$

$d_2 = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 1 \;\; \lambda/\mu \;\; 1 \;\; \lambda/\mu \;\; 1 \;\; 1 \;\; 1 \;\; 1]$

$$\overline{B}_0 = \frac{1}{\lambda\mu} \, d_1 \widetilde{B}_0 d_2$$

In the present application, the effect of aerodynamic damping, on the flutter characteristics, has been neglected. Hence eqn. (1) with the assumption $\mathbf{q} = \mathbf{q}e^{\bar{\lambda}t}$ reduces to

$$[\mathbf{K}_E + C_0\mathbf{B}]\,\mathbf{q} + C_2\bar{\lambda}^2 [\mathcal{M}]\,\mathbf{q} = 0 \quad . \quad (7)$$

or in the form of conventional eigenvalue problem it may be written as

$$[\mathbf{K}_E + C_0\mathbf{B}]^{-1} [\mathcal{M}]\,\mathbf{q} = \zeta^2\mathbf{q} \quad . \quad . \quad (8)$$

where

$\zeta = \eta_R + i\mu_I$ is the required complex eigenvalue.

The natural frequency CD may he obtained from eqn. (H), by equating C_0 to zero us a particular case. The natural frequency may be related to the conventional constant k by

$$k = 4\mu_1 = \omega l^2 \sqrt{(\rho_m h/D)}. \quad . \quad . \quad (9)$$

liquation (8) may he solved for various value* of C_0, •the aerodynamic parameter. The flutter conditions may be obtained by studying the variation of eigenvalues with the gradual increase in C_0. It; may be observed that as C_0 is increased, two sets of eigenvalues approach each other and after a certain value of C_0, they become complex conjugates. The particular value of C_0, at which the two frequencies coalesce gives the critical value.

Results and Conclusions

For the numerical calculations, n panel with simply-supported end conditions has been chosen and is divided into live elements in the streamwise direction and four elements In the span-wise direction, (Fig. 1). The present results refer to three different aspect ratios (0·5, 1·0 and 2·0) and for each case three skew angles, namely, 90°, 75" and 60° have been considered. However, any boundary

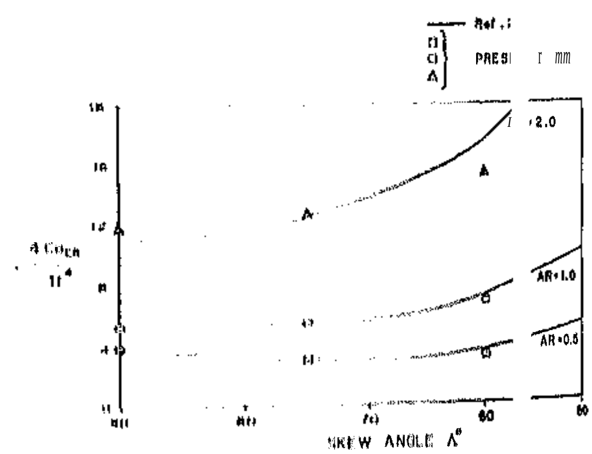

Figure 2. Variation of critical aerodynamic parameter with angle of skew.

condition with any aspect ratio and skew angle may be accomplished with little effort in the computer program.

The results of the present flutter analysis are compared with the other available results of ref. 2 and are represented in Fig. 2 in the form of a graph showing the variation of critical dynamic pressure with angle of skew. Fig. 3 shows the variation of the first two natural frequencies with the angle of skew. The computed natural frequencies compared with the known values of the rectangular plates[7,8] are in excellent agreement and, hence, the number of discrete elements, chosen for the numerical computation, may be considered to bo sufficient for the purpose.

The comparison of flutter results show that there is good agreement between the present finite element approach and that of conventional method of ref. 2 employing double Fourier sine series, for the aspect ratios 0·5 and 1·0. But there is slight disagreement in the case of aspect ratio 2·0. In the absence of exact theories and realistic experiments, it is difficult to comment at this stage about the relative merits of the results. Nevertheless, the finite element method has proved its superiority over other methods in its generality and applicability to accommodate practically any boundary conditions, including structural discontinuities, if any. No convergence studies have been attempted in this analysis. However, the application of the kinematically consistent quantities have proved to be satisfactory in other dynamic problems as illustrated in ref. 9 and other works relating to finite element approach.

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APPENDIX

The use of natural modes, suggested by Argyris, which include both straining and rigid body modes in the matrix displacement approach, has proved in applications to both linear and non-linear fields highly superior compared to other methods which make use of, say, the kinematic modes due to unit displacements[9]. Natural mode approach yields, with a minimum of algebra, concise expressions for the elemental stiffnesses compared to other approaches which become cumbersome and lead to complicated expressions. Also in the evaluation of kinematically consistent lumped masses, the natural mode technique is again more successful and, hence, the same technique is further extended in the present application for deriving the kinematically consistent aerodynamic influence coefficient matrices.

The deformed shape of an element is uniquely described in terms of its nodal displacements p and the state of deformation of the element is expressed as a combination of nine straining modes and three rigid body modes ρ'.

Natural mode vector

$$\rho' = \{\rho_N \rho_0\}$$

where ρ_N is a (9×1) straining mode vector and ρ_0 is a (3×1) rigid body mode vector.

The relation between the natural and kinematic mode

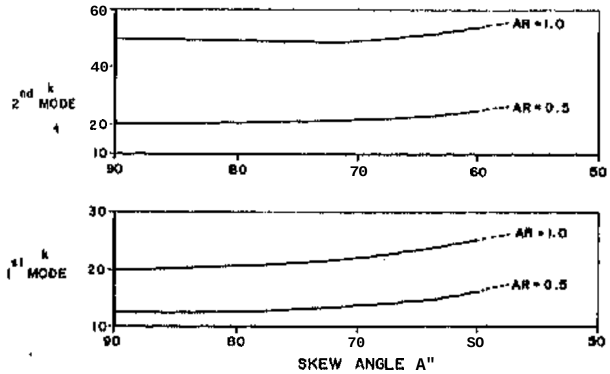

Figure 3. Frequency constant (k) vs. skew angle.

vectors may be written as

$$\rho' = a_0 \rho$$

where a_0 is a transformation matrix[9].

The displacement at any point outside the element may be expressed as

$$w = [\omega_n \omega_0] \begin{Bmatrix} \rho_N \\ \rho_0 \end{Bmatrix} = \overline{\omega}\,\rho' = \overline{\omega}\, a_0 \rho$$

or $w = \Omega\rho$

where

$$\omega_n' = \begin{bmatrix} \frac{1}{16\lambda}(1-\xi^2) \\ \frac{1}{16\mu}(1-\eta^2) \\ -\frac{1}{16\lambda}\xi(1-\xi^2) \\ -\frac{1}{16\mu}\eta(1-\eta^2) \\ -\frac{1}{32\lambda}\eta(1-\xi^2)(3-\eta^2) \\ -\frac{1}{32\mu}\xi(3-\xi^2)(1-\eta^2) \\ \frac{1}{32\lambda}\xi\eta(1-\xi^2)(3-\eta^2) \\ \frac{1}{32\mu}\xi\eta(3-\xi^2)(1-\eta^2) \\ \frac{1}{4}\xi\eta \end{bmatrix}$$

corresponding to nine straining modes.

$$\omega_0' = \begin{bmatrix} \frac{1}{4} \\ -\frac{1}{8\lambda}\xi \\ \frac{1}{8\mu}\eta \end{bmatrix}$$

corresponding to three rigid body modes.