ANALYSIS OF AXISYMMETRIC LAMINATED COMPOSITE SHELLS SUBJECT TO ASYMMETRIC LOADING
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Summary

A finite element formulation for the static analysis of a laminated composite shell of revolution with general meridional curvature, subjected to asymmetric loading is presented. The analysis uses an axisymmetric laminated shell element where the shell geometry is satisfactorily represented and higher order polynomial approximations are used for the displacement fields. The asymmetric loading problem is handled through a Fourier series representation of the applied loads and the resultant displacements. Solutions are presented for typical aerospace shell structures like a composite cone and a tangent ogive shell subjected to wind loads.

Introduction

Many aerospace structural configurations are shells of revolution. These shells are often fabricated using fibre reinforced composite materials by the filament winding process or by moulding using several layers of impregnated fabrics. Though the fundamental unit in these constructions is a unidirectional lamina, the overall laminate may be orthotropic, nearly orthotropic or generally anisotropic depending on the number of layers and individual layer orientations. Moreover, even though these shells are axisymmetric structures the aerodynamic and inertial loading on them are generally asymmetric. Because of the anisotropic material properties and asymmetric nature of the loading the analysis of these type of shells is usually quite complex.

Most of the analytical formulations available in literature are limited to simple geometrical configurations like cylindrical and conical shells with homogeneous isotropic or orthotropic material properties. The influence of material anisotropy in axisymmetric shells with asymmetric loading has been dealt by Pandovan and Lestingi [1,2]. These authors employ a multisegment numerical integration technique for the solution of shell equilibrium equations using a finite exponential Fourier transform.

The finite element method of analysis provides an alternate simpler method of solution for shells with complex geometries, arbitrary loadings, general boundary conditions and anisotropic material characteristics. Although shells of revolution can be analysed using general shell elements like high precision triangular laminated anisotropic shallow shell element [3], the use of axisymmetric shell elements is simple and attractive for a large class of practical problems [4-8]. However, finite element formulations that take into account orthotropic and anisotropic material properties are limited in number. Reference [9] uses a truncated cone type element (orthotropic) to represent even arbitrary meridional curvatures. Pandovan [10] uses a quasi-analytical finite element procedure with complex Fourier transforms for displacement and force representation for solving the resultant complex displacement equilibrium equations for anisotropic axisymmetric shells.

The analysis presented in this paper uses a finite element procedure based on matrix displacement method. The element employed is a refined axisymmetric shells element specially formulated for the analysis of laminated composite shells of revolution [11]. The asymmetric loading problem is handled through a Fourier series representation of the loads and displacements using a cosine and a sine series for the symmetric and the antisymmetric components respectively. When the shell is isotropic or orthotropic the force-displacement field equations for each harmonic in the series are uncoupled and can be solved separately. However, for a general anisotropic shell, the generalized coordinates corresponding to the sine and cosine series for a given harmonic are coupled and hence will have to be considered together. This leads to doubling of the size of the stiffness matrix and the number of force-displacement equations to be solved. Fortunately for a large number of practical laminated structures, the coupling is zero (i.e. orthotropic) or its effect is small. In such cases, effect of the coupling terms can be neglected and solutions obtained for the sine and cosine components in each harmonic separately.

The finite element method presented in this paper has the following capabilities:

1. It can represent a shell with general meridional curvature.
2. Variation of thickness and material properties along the meridian.
3 Variation of applied loads along the meridian.

4 Loading can be asymmetric.

2 Analysis Procedure

2.1 Element Description

The element employed is a refined axi-symmetric laminated composite shell of revolution element with two nodal circles and seven degrees of freedom per node. A brief description of the element and the development of this formulation are given below.

![Axiymmetric Shell Element - Geometry and Coordinate System](image)

**FIG 1. Axisymmetric Shell Element - Geometry and Coordinate System**

The element has seven degrees of freedom per node which are the displacement components \( u, v, w \), first and second derivatives \( w' \) and \( w'' \) of the circle midplane component \( u \) and \( v \). The out of plane component \( w \) and its first and second derivatives \( w' \) and \( w'' \) (subscripts \( v \) and \( w \) denote first and second derivatives respectively with respect to \( r \)).

The components \( u, v, w, w', w'' \) in the series represented by equations (5) describe the variation in the meridian direction and are interpolated in terms of the generalized displacements at each nodal circle using third order polynomials for \( u \) and \( v \) and fifth order polynomial for \( w \).

These relations can be expressed in the general matrix form:

\[
[A^u] \cdot i = i \{u^1\} \quad i = 0,1,2,..,N_v \quad (5)
\]

\[
[A^v] \cdot i = i \{v^1\} \quad i = 0,1,2,..,N_v \quad (6)
\]

where \( i \{u^1\} \) and \( i \{v^1\} \) are matrices of generalized coordinates, \( i \{u^1\} \) and \( i \{v^1\} \) matrices of generalized nodal degrees of freedom, and \( i \{u^1\} \) a matrix of polynomials in \( i \).

\[
\begin{align*}
[A^u] &= \begin{bmatrix}
A_{11} & A_{12} & A_{13} \\
A_{21} & A_{22} & A_{23} \\
A_{31} & A_{32} & A_{33}
\end{bmatrix},

\{u^1\} &= \begin{bmatrix}
u_1 \\
v_2 \\
v_3
\end{bmatrix}
\end{align*}
\]

and

\[
\begin{align*}
[A^v] &= \begin{bmatrix}
B_{11} & B_{12} & B_{13} \\
B_{21} & B_{22} & B_{23} \\
B_{31} & B_{32} & B_{33}
\end{bmatrix},

\{v^1\} &= \begin{bmatrix}
v_1 \\
v_2 \\
v_3
\end{bmatrix}
\end{align*}
\]

and similarly for \( i \{u^1\} \) and \( i \{v^1\} \). Matrix \( \{u^1\} \) is given in Appendix 1.

**Appendix 1**

**Matrix \( \{u^1\} \) in the relations \( i \{u^1\} = i \{u^1\} M \)**

**Non - Zero Elements**

\[
\begin{align*}
X(1,1) &= X(1,1) & T^r & \varphi \\
X(1,2) &= X(1,2) & T^r & \varphi \\
X(1,3) &= X(1,3) & T^r & \varphi \\
X(1,4) &= X(1,4) & T^r & \varphi
\end{align*}
\]
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This linear displacement relations for a shell of revolution are expressed as [12],

\[ \begin{align*}
E_g & = \nu_{11} \cdot \omega \phi_{11} \\
0 & = \frac{1}{\nu_{22}} \left( v \cdot \nu \cdot \phi_{22} + w \cdot \omega \cdot \phi_{22} \right) \\
I_{th} & = \frac{1}{\nu_{22}} \cdot \nu_{22} - \frac{1}{\nu_{22}} \cdot \nu_{22} \\
K_1 & = -\frac{2}{\nu_{22}} \cdot \omega \phi_{22} \\
K_2 & = -\frac{2}{\nu_{22}} \cdot \nu_{22} \cdot \phi_{22} \\
K_3 & = -2 \phi_{22} \\
\end{align*} \]

where superscripts \( 1 \) and \( 2 \) denote first and second derivative with respect to \( s \) and \( \phi \), respectively.

\[ R = \frac{1}{\nu_{22}} \left( \nu_{22} \cdot \nu_{22} \right) \\
\]

\[ \begin{align*}
& E^{1} \cos J \theta \ \\
& E^{2} \sin J \theta \ \\
& E^{3} \cos J \theta \ \\
& E^{3} \sin J \theta
\end{align*} \]

The coefficients in the above series are given by the following matrix equations.

\[ \begin{align*}
\begin{bmatrix}
E^{1} \\
E^{2} \\
E^{3}
\end{bmatrix} &= \begin{bmatrix}
Y_1 \ \\
Y_2 \ \\
Y_3
\end{bmatrix} \\
A^{J}_n &= J = 0, 1, 2, 3, ..., N_c
\end{align*} \]

\[ \begin{align*}
\begin{bmatrix}
E^{1} \\
E^{2} \\
E^{3}
\end{bmatrix} &= \begin{bmatrix}
Y_1 \ \\
Y_2 \ \\
Y_3
\end{bmatrix} \\
A^{J}_n &= J = 1, 2, 3, ..., N_b
\end{align*} \]

Substituting expressions (5) and (6) in equations (11) and (12) respectively, the strain components can be directly expressed in terms of nodal variables \( q \) and \( q' \).

2.4 Strain Energy and Stiffness Matrices

The elastic strain energy \( U \) stored in an element of a laminated composite shell during deformation is given by

\[ U = \frac{1}{2} \int_{\theta = 0}^{2\pi} \left( E^{1} \cos J \theta + E^{2} \sin J \theta ight) \left( K^{1} \cos J \theta + K^{2} \sin J \theta \right) \]

where \( A^{J} \) and \( K^{J} \) are given in Appendix III.

Substituting for \( E \) and \( K \) in terms of nodal variables \( q \) and \( q' \), the expression for \( U \) can be written in terms of the nodal degrees of freedom \( q \) and \( q' \) as follows.
where $[K]^i$, $[K]^s$ and $[K]^a$ are the element elastic stiffness matrices corresponding to the constant part, symmetric part of the $j$th harmonic and anti-symmetric part of the $j$th harmonic respectively in the Fourier series representation of the displacements, and $[K']^i$ is a coupling stiffness matrix representing the coupling in the $j$th harmonic between the two series in the displacement functions. Expressions for $[K']^i$, $[K']^a$, $[K']^s$ and $[K']^p$ are as follows.

\[
[K']^i = 2\pi \int r \left( \begin{array}{cc}
\frac{\partial u_i}{\partial r} & 0 \\
0 & \frac{\partial u_i}{\partial r}
\end{array} \right) dr,
\]

\[
[K']^s = \int r \left( \begin{array}{cc}
\frac{\partial u_s}{\partial r} & 0 \\
0 & \frac{\partial u_s}{\partial r}
\end{array} \right) dr,
\]

\[
[K']^a = \int r \left( \begin{array}{cc}
\frac{\partial u_a}{\partial r} & 0 \\
0 & \frac{\partial u_a}{\partial r}
\end{array} \right) dr,
\]

\[
[K']^p = \int r \left( \begin{array}{cc}
\frac{\partial u_p}{\partial r} & 0 \\
0 & \frac{\partial u_p}{\partial r}
\end{array} \right) dr.
\]

Matrices $[A]$, $[B]$ and $[D]$ are derived from matrices $[A]$, $[B]$ and $[D]$ respectively by replacing elements $A_{16}$, $A_{26}$ in $[A]$, $B_{16}$, $B_{26}$, $D_{16}$, $D_{26}$ in $[D]$ by $A_{16}$, $A_{26}$, $B_{16}$, $B_{26}$, $D_{16}$ and $D_{26}$ as zero.
element, for a given harmonic number $J$ in the displacement functions, the corresponding generalized forces $\{Q^J\}$ and $\{Q'^J\}$ are functions of both $\{q\}$ and $\{q'\}$. However, when $A_{16}$, $A'_{26}$, $B_{17}$, $B'_{27}$, and $D_{18}$ and $D'_{28}$ are either zero or small, or its elements are small compared to those of $[K]$ and $[K']$, and can be neglected.

For these cases $\{Q^J\}$ and $\{Q'^J\}$ are uncoupled and are given by

$$\{Q^J\} = [K^J] \{q^J\}$$

$$\{Q'^J\} = [K'^J] \{q'^J\}$$

(18)

It is clear from eqns (17) and (18) that when the coupling effects are significant, the generalized coordinates $\{q^J\}$ and $\{q'^J\}$ corresponding to the cosine and sine series should be considered together. As a result, the number of degrees of freedom per element, the size of the stiffness matrix and consequently the number of final force displacement equations to be solved are twice those for the case when the coupling is zero or negligible (Eqn. 16).

2.5 Stress Resultants

For arbitrarily laminated composite shells, the membrane and bending stress resultants (Fig. 2) are related to the reference surface strains and curvature changes by the following matrix equation (13).

![Fig. 2. Membrane and Bending Stress Resultants](image)

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{bmatrix} E \\ k \end{bmatrix}$$

(19)

Where

$$\begin{bmatrix} N \\ M \end{bmatrix} = \begin{bmatrix} N_5 \\ N_6 \\ N_9 \\ M_5 \\ M_6 \\ M_9 \end{bmatrix}$$

(20)

combining eqns. (19) and (10), each of the stress and moment resultants can be expressed as follows.

$$\begin{align*}
N_s &= N_5 + \sum_{J=1}^J N_5^J \cos 2\pi J \sin \theta + \\
&\quad \sum_{J=1}^J N_5^J \sin 2\pi J \cos \theta \\
N_c &= N_6 + \sum_{J=1}^J N_5^J \cos 2\pi J \cos \theta + \\
&\quad \sum_{J=1}^J N_5^J \sin 2\pi J \sin \theta \\
M_s &= M_5 + \sum_{J=1}^J M_5^J \cos 2\pi J \cos \theta + \\
&\quad \sum_{J=1}^J M_5^J \sin 2\pi J \sin \theta \\
M_c &= M_6 + \sum_{J=1}^J M_5^J \cos 2\pi J \sin \theta + \\
&\quad \sum_{J=1}^J M_5^J \sin 2\pi J \cos \theta
\end{align*}$$

(21)

and similarly for $M_{s0}$, $M_{c0}$, $M_{s1}$ and $M_{c1}$. Here,

$$\begin{bmatrix} N_5^J \\ E_5^J \end{bmatrix} = \begin{bmatrix} A^* & B^* \\ B^* & D^* \end{bmatrix} \begin{bmatrix} E^J \\ k^J \end{bmatrix}$$

(22)

2.6 Kinematically Consistent Loads

The externally applied loads on the element are first converted into a set of kinematically equivalent loads consistent with the element degrees of freedom using the principle of virtual work. The procedure for external normal pressure whose distribution may be asymmetric around the circumference is given here. The pressure $p$ is first decomposed into a series of harmonic functions around the circumference as follows.

$$p = p_0 \sum_{J=1}^J N_5^J \cos 2\pi J \sin \theta + \sum_{J=1}^J p_0^J \sin 2\pi J \cos \theta$$

(23)

The virtual work done by the pressure-
virtual work = \int_0^{2\pi} \mathbf{w}^T \mathbf{D} \mathbf{f} \, d\theta \cdot \mathbf{w} \quad (26)

On substituting for \( \mathbf{p} \) and \( \mathbf{w} \) from eqns. (24) and (4) respectively and integrating between the limits \( 0 \) to \( 2\pi \), the expression for the virtual work can be expressed as

Virtual Work = \sum_{i,j} \mathbf{N} \mathbf{p}_{ij} \int_0^{2\pi} \mathbf{w}_{ij}^T \mathbf{w} \, d\theta \quad (29)

whom \( \mathbf{p}_{ij} \) is 2n\( \times \)2n matrix \( f(\theta) \) and \( \mathbf{w}_{ij} \) is n\( \times \)1 column of \( f(\theta) \) for each element. Eqns. (11) and (12) then give the displacement vectors. Hence, evaluation of the elemental nodal variables for each element, Eqns. (11) and (12) then give the elemental nodal variables and evaluation of the elemental stiffness matrix \( \mathbf{K}_e \). Thus the elemental nodal variables are obtained using eqn. (13) for \( \mathbf{K}_e \) in a typical co-opal example, where the elemental stiffness matrix \( \mathbf{K}_e \) is obtained from the elemental nodal variables. In doing so and combining the elemental nodal variables, similar expressions can also be derived for generalized load vectors corresponding to externally applied inplane loads.

2.7 Assembly and Solution

In the small displacement analysis each harmonic component of the applied load is related only to the nodal displacements of the same harmonic through a corresponding stiffness matrix. Thus there is no coupling between different harmonics (that is, between \( i \)th and \( j \)th harmonics). In practice therefore, there are only as many harmonic terms in the displacement series as it is necessary for an accurate description of the applied load. Hence it is possible to assemble the stiffness matrix of the complete structure for each harmonic \( j \) and find the solution separately. When all the element matrices are assembled and boundary conditions incorporated the resulting discrete system field equations can be represented in the following form:

\[
\mathbf{K}_e \mathbf{p} = \mathbf{f}(\theta) \quad (26)
\]

where \( \mathbf{p} \) is the vector of unknown generalized nodal displacements composed of sub-vectors \( \mathbf{p}_j \) at the harmonic nodal number \( j \). Taking into account the boundary conditions, \( \mathbf{K}_e \mathbf{p} = \mathbf{f}(\theta) \). The structural stiffness matrix \( \mathbf{K}_e \) is the vector of generalized harmonic, consistent loads.

Once the global nodal displacement \( \mathbf{p}_j \) corresponding to the \( j \)th harmonic are determined from the solution of equation (26), it is possible to go back to the elemental level by evaluating the sub-vectors \( \mathbf{p}_j \) from the elemental nodal variables \( \mathbf{u}_j \) and \( \mathbf{v}_j \) and \( \mathbf{w}_j \) for each element. Eqns. (11) and (12) then give the elemental nodal variables and evaluation of the elemental stiffness matrix \( \mathbf{K}_e \). Hence, the elemental nodal variables are obtained using eqn. (13) for \( \mathbf{K}_e \) in a typical co-opal example, where the elemental stiffness matrix \( \mathbf{K}_e \) is obtained from the elemental nodal variables. In doing so and combining the elemental nodal variables, similar expressions can also be derived for generalized load vectors corresponding to externally applied inplane loads.

5 Applications to Practical Problems

The method presented in this paper is verified by applying it to a loading case in a computer using \( \text{SHEAR} \) IV language. \( \text{SHEAR} \) IV is a general purpose computer program for structural analysis. The three displacement and load vectors \( \mathbf{p}_j \) are assembled using numerical integration in MATLAB. The three displacement and load vectors \( \mathbf{p}_j \) for each harmonic \( j \) are solved using the Bielecki square root de-Moore-Penrose algorithm for fixed boundary conditions.
Solutions are obtained for a fibre reinforced composite cone and a tangent ogive shell, subjected to asymmetric wind loads. These are described below.

3.1 Cone

Fig 5a shows geometry of the cone. It is assumed to be fabricated using layers of resin impregnated woven glass cloth. Typical properties of a woven glass laminate used in the analysis are given in Table 1 [14]. The circumferential wind pressure distribution, assumed to be the same at all stations along the length, is shown in fig 6a and its Fourier series representation is also indicated. For analytical purposes the L and T directions of the fibre lay-ups are assumed to be coincident with the meridional and circumferential directions.

The cone is assumed to be rigidly supported at the base and free at the tip. A total of 30 elements are used in the finite element idealization, the nodes being closer near the base to take care of sharp variations due to bending effects. Typical results for displacement and stress resultant variations for cones with wall thickness varying from 1.0 to 5.0 mm are given in fig 8 to 11. The parameters presented in all the figures correspond to a unit value of wind dynamic pressure q.

Fig 6a shows the variation of radial deflection along the windward generator (i.e. $G_0$) , and Fig 8b shows its circumferential variation at the free end. The deflection is inwards (negative) at the windward side, its maximum value occurring at the free end, but becomes outwards (positive) at the leeward side (i.e. for $0$ greater than about $7.8^\circ$).
Table 1

Mechanical Properties of Woven glass Laminate (Ref.14)

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resin content</td>
<td>31%</td>
</tr>
<tr>
<td>$F$</td>
<td>$25.993 \times 10^6$ kN/m$^2$</td>
</tr>
<tr>
<td>$E$</td>
<td>$19.995 \times 10^6$ kN/m$^2$</td>
</tr>
<tr>
<td>$G_{LT}$</td>
<td>$4.895 \times 10^6$ kN/m$^2$</td>
</tr>
<tr>
<td>$\mu_{LT}$</td>
<td>0.1644</td>
</tr>
<tr>
<td>$\sigma_{LU}$</td>
<td>309.24 MN/m$^2$</td>
</tr>
<tr>
<td>$\sigma_{Tu}$</td>
<td>216.36 MN/m$^2$</td>
</tr>
<tr>
<td>$T_{LT}$</td>
<td>21.72 MN/m$^2$</td>
</tr>
</tbody>
</table>

FIG 6(a). Circumferential Pressure Distribution of Cone

FIG 6(b). Circumferential Pressure Distribution of the Ogive

The variations of stresses and moment residuals $N_x, M_x, N_y,$ and $M_y$ along the windward generator are shown in figures 9a, b and 10a, b respectively. $N_x, M$ and $N_y$ assume their largest magnitude near the base whereas the maximum value of $M_y$ occurs near the free end and the location of this maximum shifts away from the free end as the wall thickness increases.
Fig 9(a). $N_\theta$ Variation Along Windward Generator ($\theta=0^\circ$)

Fig 9(b). $M_\theta$ Variation Along Windward Generator ($\theta=0^\circ$)

Fig 10(a). $N_\theta$ Variation Along Windward Generator ($\theta=0^\circ$)

Fig 10(b). $M_\theta$ Variation Along Windward Generator ($\theta=0^\circ$)

Fig 11(a) and b depict the circumferential variation of $M_\theta$ and $N_\theta$ respectively at the fixed end. $N_\theta$ is tensile near the region $\theta = 0^\circ$, but becomes compressive for $\theta$, greater than about 55°, whereas the bending stress resultant $M_\theta$ remains positive all round the circumference causing tensile bending stresses in the outer fibres and compressive stresses in the inner fibres.

3.2 Tangent Ogive Shell

Fig 5a shows the geometry of the tangent ogive. As in the case of the cone example, the shell is assumed to be fabricated using layers
of resin impregnated woven glass cloth, the typical properties of which are given in Table 1. Once again the L and T directions of the fibre layup are assumed to coincide with meridional and circumferential directions.

The external pressure acting on the shell is assumed to be of the form $q_0 (X) \cos \theta + q_1 (X) \sin \theta$. The variation of the multipliers $q_0 (X)$ and $q_1 (X)$ along the length of the generator is shown in Fig. 6. The Fourier expansion of the pressure distribution can readily be written as:

$$ p_0 (X, \theta) = \frac{1}{2} p_0 (X) + \sum_{j=1}^{\infty} \left( p_{j1} (X) \cos j\theta + p_{j2} (X) \sin j\theta \right) $$

where $p_0 = \frac{1}{3} (q_1 - q_2)$, $p_{j1} = \frac{1}{2} q_j^1$, $p_{j2} = \frac{1}{2} q_j^2$, and $p_{j3} = \frac{1}{2} q_j^3$.

The shell is assumed to be rigidly fixed at the base and free at the tip. 50 elements are used to idealize the shell.

Fig. 12 gives the radial displacement $u_1$ along the generators. It is maximum at $\theta = 0$ and $\theta = \pi$, decreasing maximum value in about 0.01 mm at the tip. At the tip, $u_1$ is maximum near the maximum of the generator is shown in Fig. 6. The maximum value of $u_1$ is near the maximum at about 0.01 mm from the tip.

Fig. 13 also gives the distribution of various stress resultants $N_1, N_3$, and $N_9$ along the generators. $N_1$ at $\theta = 0$ will be tensile, near the tip and becomes compressive at the fixed end. $N_3$ at $\theta = \pi$ remains tensile throughout the generator and becomes compressive at the fixed end. $N_9$ remains tensile and again compressive at $\theta = \pi$. $N_9$ shows the maximum of $u_1$ at the root as well as the tip. The graph shows the distribution of bending stress resultants $M_1$, $M_3$, and $M_9$ along the generators. $M_1$ reaches its maximum value at the fixed end, $M_3$ at the tip, and $M_9$ in the middle of the shell. $M_1$ maximum occurs approximately at 0.01 mm from the tip. At the fixed end $M_3$ causes compressive bending stress, and $M_9$ in the middle of the shell. $N_9$ is maximum in the middle of the shell.

As can be seen from Fig. 1, magnitude $u_1$ is comparatively less than that of $q_0$ and the influence of the term $q_1 \cos \theta$ in the expression for pressure is seen to be predominant in all the distributions.
4 Conclusions

An axisymmetric laminated shell finite element is used for the static analysis of laminated composite shells of revolution subjected to axisymmetric loading. A Fourier expansion is used to represent the circumferential variation of the loading and the resultant displacements. The examples considered are a cone and a tangent ogive shell made of glass reinforced composite and subjected to wind pressure loading. The analysis can also be used for the case of inertial and thermal loading.

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