The structure of laminar flow in a three-dimensional driven cavity

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Abstract: We present some details of the steady laminar flow fields that are generated in a fluid filled cavity by the motion of one of the faces. The flow fields have been obtained by the numerical solution of the Navier-Stokes equations in the Reynolds number range $0 < Re < 1000$. The streamline patterns clearly indicate the very complicated nature of the eddy structures and their dependence on Reynolds number. Further work is currently in progress to extend the computations to higher Reynolds numbers in order that transition and turbulence in the cavity may be studied.

1. Introduction

The geometry we consider is shown in Fig. 1. The motion in a fluid filled rectangular parallelepiped is driven by the motion of the wall $x = 0$ in the y-direction. We normalize all lengths by l_y, the y-width of the parallelepiped, velocities by v_0, the speed of the moving wall, time by l_y/v_0 and pressure by ρv_0^2. The Reynolds number is then $v_0 l_y/\nu$. Our aim here is then to determine the flow structure in the cavity for all Reynolds numbers for which the flow is steady and laminar in the cavity.

Whereas the 2-D problem has been studied in great detail the 3-D problem does not seem to have gained as much attention. This is quite understandable since the computational requirements increase dramatically with increased dimension. The most important 3-D studies so far are due to the Stanford group (Koseff and Street 1984, Prasad and Koseff 1989) and the Tokyo group (see Iwatsu *et al.* 1989 for example). These studies have clearly established the complex 3-D nature of the flow field with, in general, upstream and downstream vortices and corner vortices in addition to the main eddy in the cavity (see Fig. 1).

This paper presents some of the results of our own computations on such flows in the steady laminar range. For the details of the method used see Shankar (1993) and Deshpande (1993).

2. The laminar flow fields

When *Re* is very small the flow is determined by Stokes' equation, *i.e.*, where the inertial terms are absent. In this range the 2-D problem can be easily solved semi-analytically (see Shankar 1993). Figure 2 shows the streamline patterns for a square cavity and a cavity of depth 5, It can be shown in general that an infinite sequence of corner eddies of decreasing strength occupy the lower corners of the cavity. Our studies have shown that as the depth increases the corner eddies merge to form the secondary and later primary eddies (Fig. 3).

For 3-D flows the flow fields have to be determined by directly solving the Navier-Stokes equations. For all the computations reported here, a grid of size 24 x 24 x 24 was used. Figure 4(a) shows the streamline pattern in the symmetry plane ($z = 0.5$) for $Re = 0.1$; this is similar to the result of the 2-D Stokes calculation. At $Re = 100$ the situation has changed in that the main eddy centre is no longer on the mid-y plane (Fig. 4b).

With increasing Reynolds number the flow field becomes highly 3-dimensional with large asymmetries with respect to the mid-y plane (Fig. 5); the lower corner eddy increases in size while an upper corner eddy is formed. Some particle paths at a Reynolds number of 800 are shown in Fig. 6. Note especially that in 3-D flow fields closed streamlines are likely to be rare.

3. Conclusions

The steady laminar flow field in a driven rectangular parallelepiped has been computed for a range of Reynolds numbers. The results clearly show the complex nature of the 3-D flow field. It is planned to extend the computations to the transitional and turbulent regimes.

References

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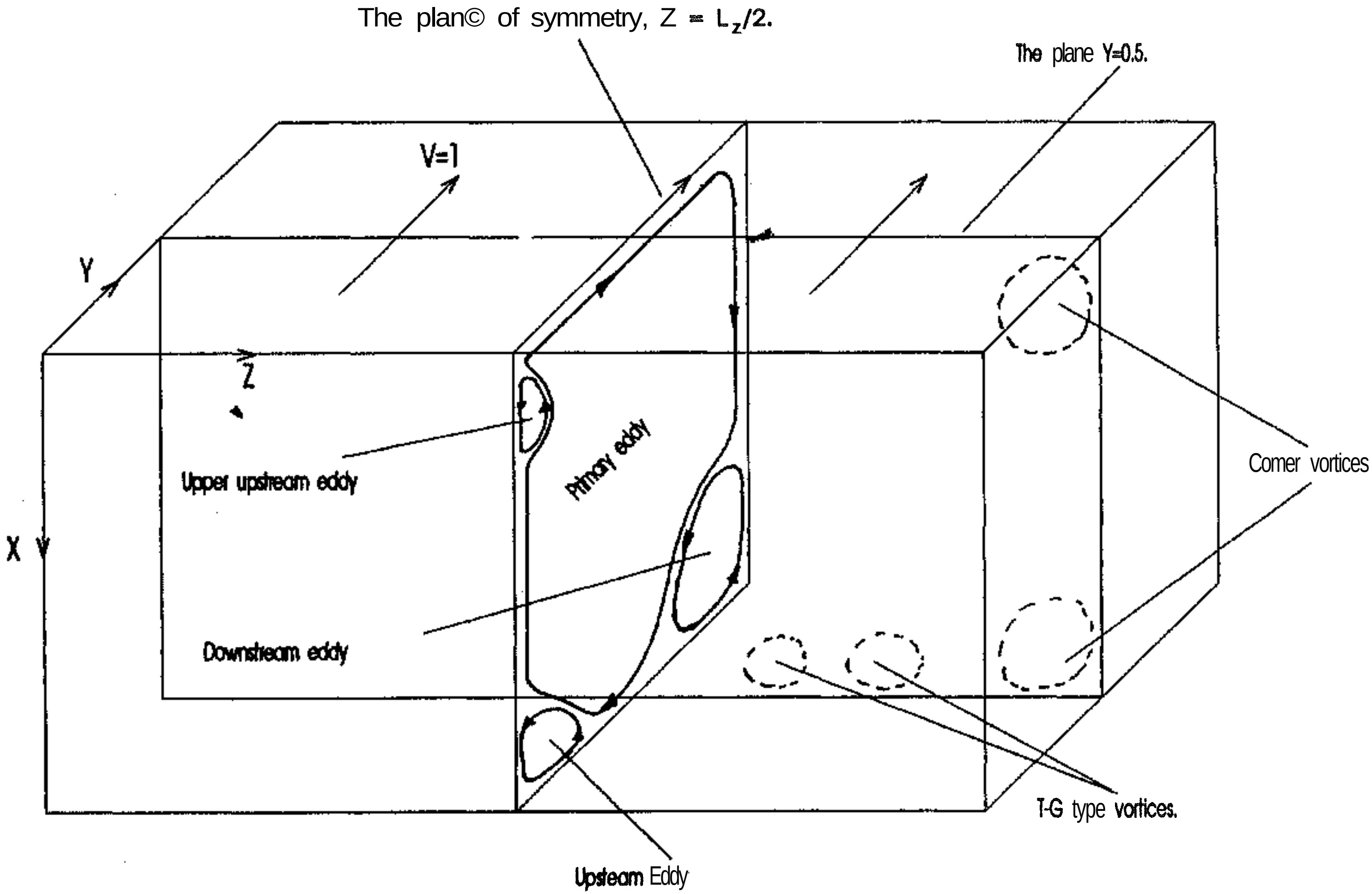


Figure 1. A schematic diagram of the flow in a rectangular parallelepiped. The nondimensional size of the box is $L_x \times 1 \times L_z$. The upper side moves at unit speed in the Y-direction.

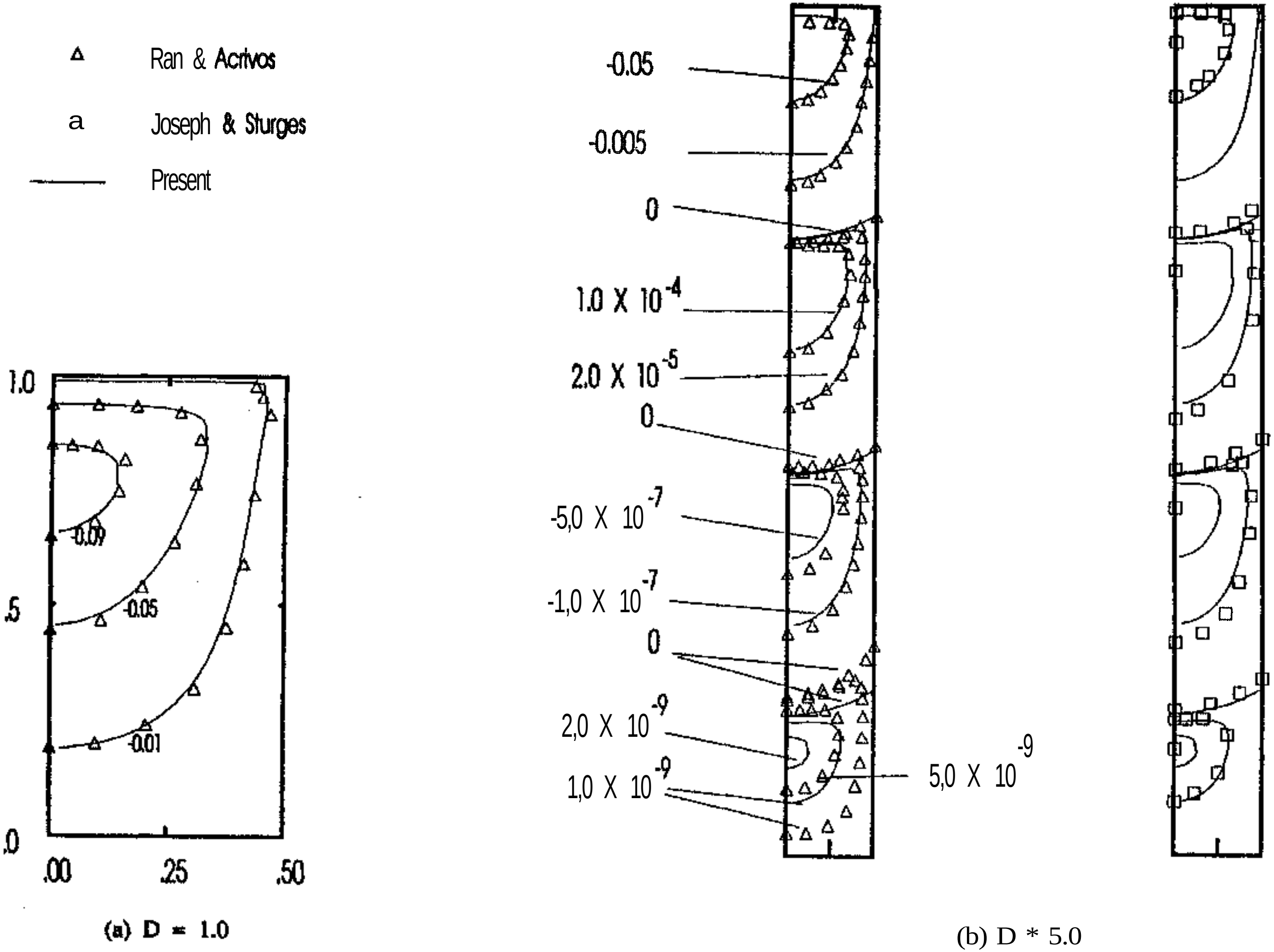


Figure 2. Streamline patterns in a square cavity and a cavity of depth 5. The numbers indicate stream function values.

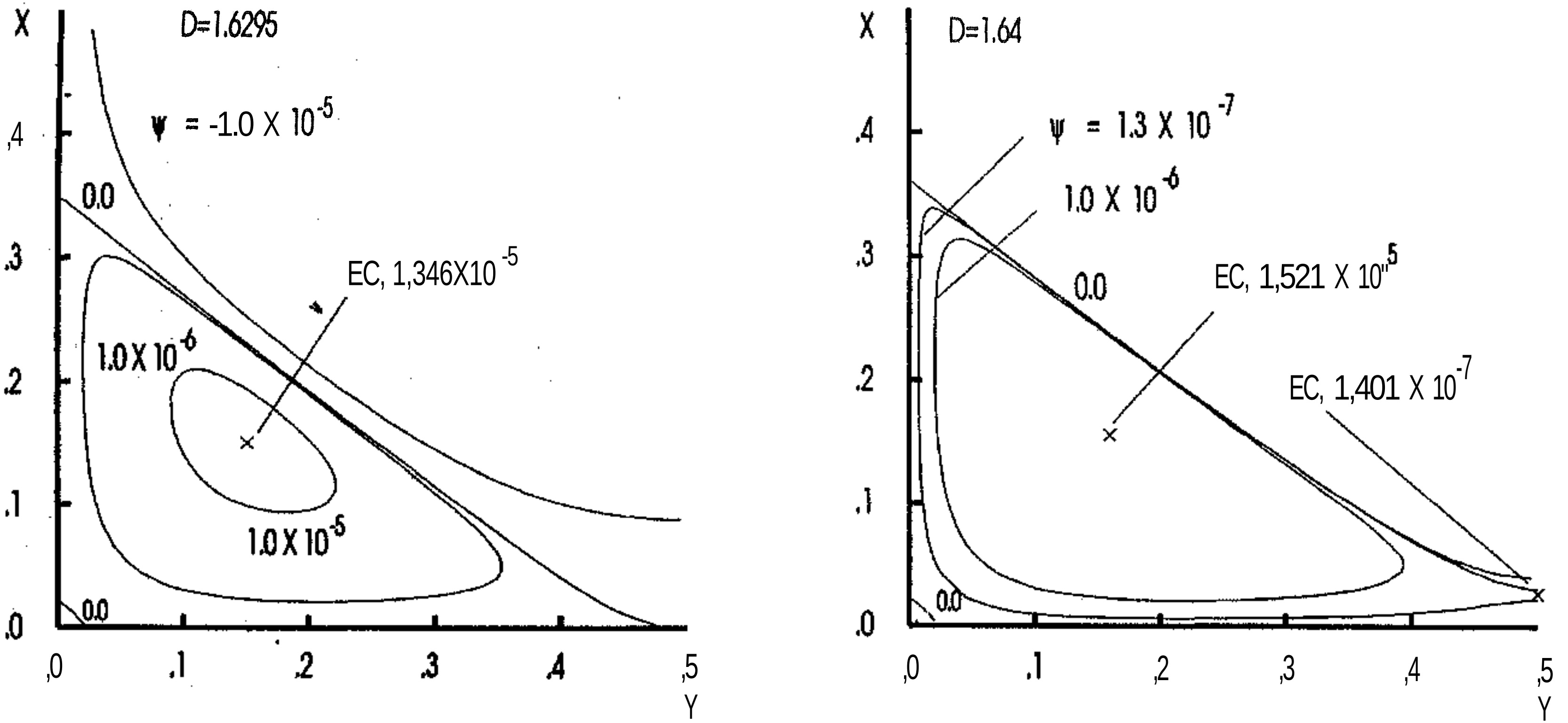
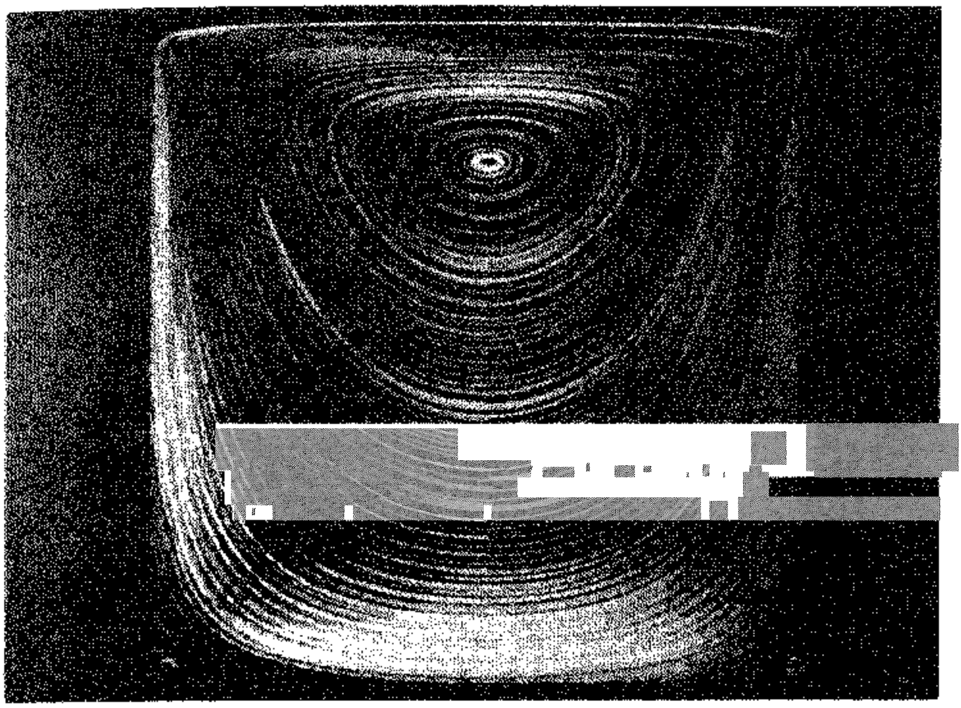
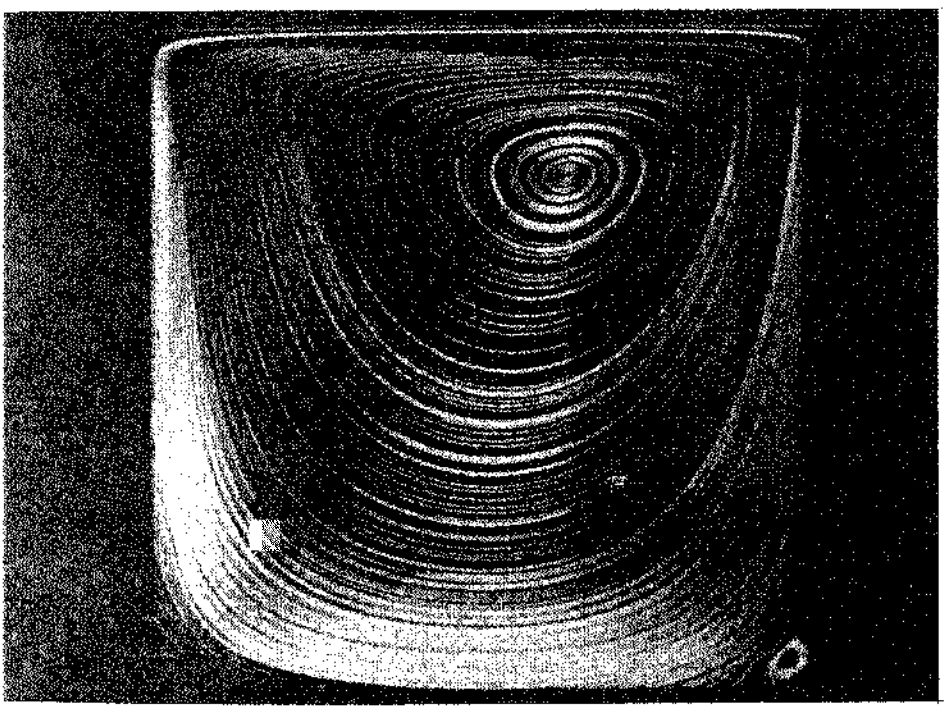


Figure 3. The evolution of the **secondary** main eddy from the primary comer eddy. EC **are** the eddy centres. Note the **eddies** at the lower **left** comer of each figure.

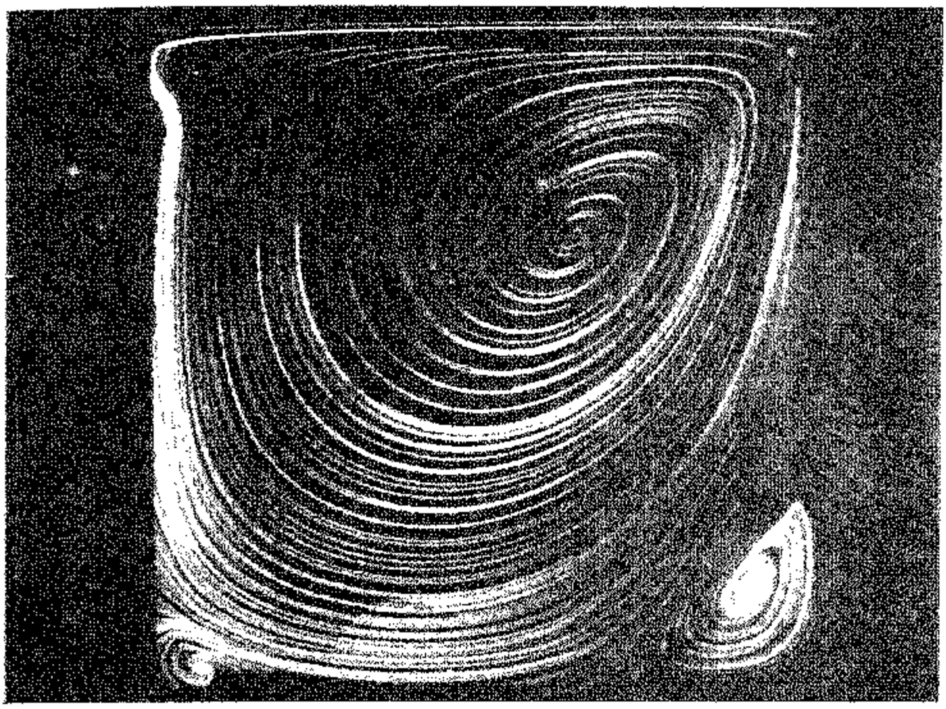


(a) Re = 0.1

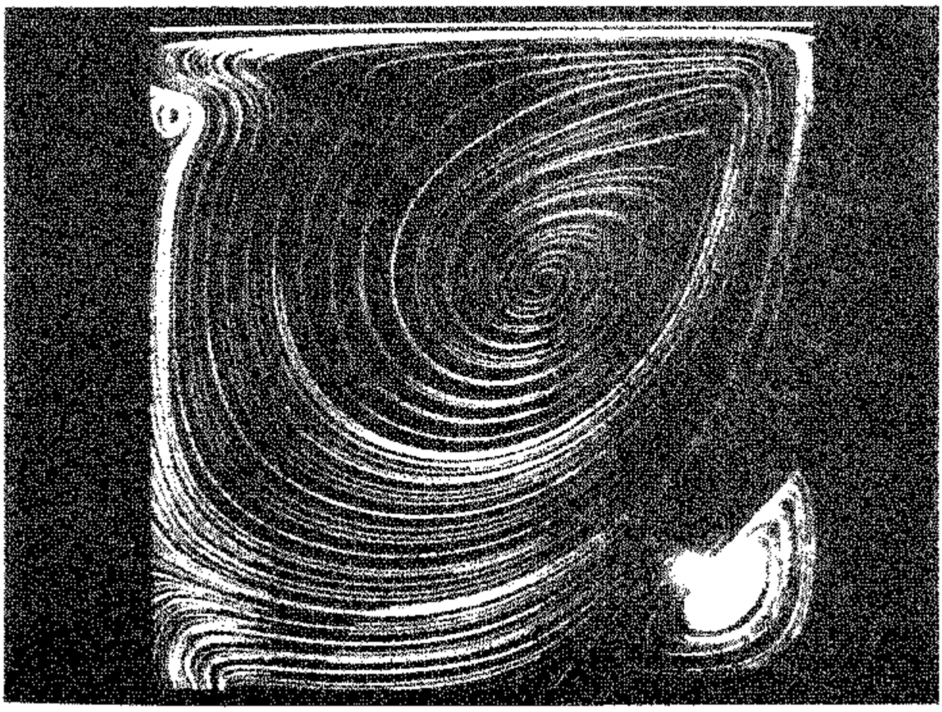


(b) Re = 100

Figure 4. Particle paths in the mid-Z plane.



(a) Re = 470



(b) Re = 1000

Figure 5. Particle paths in the mid-Z plane.