

# ESTIMATION OF LIFT AND DRAG CHARACTERISTICS OF AN AIRCRAFT FROM FLIGHT DATA

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## Abstract

*In this paper modelling and identification procedure for estimation of performance characteristics from specially conducted dynamic manoeuvres of an aircraft are described. The drag polar results from flight data are compared with the available reference values for the same aircraft.*

## Nomenclature

a	Acceleration in body axes	m/s <sup>2</sup>
b	Bias term	
$\bar{c}$	Mean aerodynamic chord	m
C	Aerodynamic coefficient for subscript	
D	drag force	kgm/s <sup>2</sup>
g	Acceleration due to gravity	m/s <sup>2</sup>
I	Moments of inertia	kgm <sup>2</sup>
$l_x, l_z$	CG distance of thrust vector in x and z axes	m
L	Lift force	kgm/s <sup>2</sup>
m	mass of aircraft	kg
p,q,r	pitch, roll and yaw rates	rad/s
$\bar{q}$	dynamic pressure	N/m <sup>2</sup>
S	reference wing area	m <sup>2</sup>
U	forward velocity	m/s
$u_0$	initial condition of forward velocity	m/s
V	true air speed	m/s
$\alpha$	angle of attack	rad
$\delta_e$	elevator deflection	rad
$\theta$	pitch angle	rad
$\theta_1$	engine thrust angle	rad
$\bar{x}$	nx1 state vector	
y	mx1 observation vector	
z	mx1 measurement vector	
v	mx1 measurement noise vector	
$\gamma$	Flight path angle	

## Subscripts, prefixes and superscripts

0	Zero or reference
x,y,z	body axes
u,v,w	small perturbations in x,y,z directions
.	dot over a quantity refers to its time derivative

## Introduction

Determination of aircraft performance parameters is an important step in the development and evaluation of modern aircraft. The drag polar estimation of the aircraft covering its full flight envelope is generally performed by 'aircraft acceleration methods' such as the total energy method for steady state flight and accelerometer method for dynamic manoeuvre. However the modern parameter estimation methods have been shown to yield good drag polar data with less restrictive manoeuvre quality. In this paper we describe various special dynamic manoeuvres which can be used to extract these parameters to be used for estimation of aircraft performance [1,2]. The flight data generated as a result of these manoeuvres are analysed using maximum likelihood method, [3]. Specifically dynamic manoeuvres like: roller coaster, windup turn and slow down, were performed on a modern fighter aircraft. These manoeuvres were analysed to generate

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complete drag polar over the full angle of attack range [4-7] and the results are compared with available reference values.

### Flight Test Experiments

Flight tests have been conducted on a fighter aircraft through series of planned dynamic manoeuvre which are described below

#### a) Roller Coaster Manoeuvre:

In this manoeuvre the aircraft is trimmed to level and steady flight at desired altitude and Mach number. The nose is slowly pulled up watching the 'g' to tract a rate of 0.1 g/second. The wings are held in level, In about 10 seconds the aircraft reaches 2g at which time the nose is pushed down to obtain a rate of -0.1g per second. In about 20 seconds the aircraft reaches Zero g. In this manoeuvre angle of attack range of 0 to about 9 degrees is covered. The quality of manoeuvre is judged by the linearity of rate of g.

#### b) Slow Down Manoeuvre:

The aircraft is trimmed to level and steady flight at desired and safe altitude with low or idle power at mach number of about 0.4. The stick is pulled very slowly so that the angle of attack (AOA) increases. It is essential to hold the aircraft altitude constant. The aircraft slows down due to increased drag and reaches the low power idling type of stall in about 25 seconds, while the AOA increases to the stall AOA. After this the aircraft is recovered to normal flight,

#### c) Windup Turn Manoeuvre:

The aircraft is initially taken to level and trimmed flight to desired speed and altitude. The aircraft is gradually banked and the load factor is increased linearly from about 1g to the maximum allowed value of g (nearly 8g). In order to do so, the bank angle is progressively increased from zero to nearly 80 deg. While pulling the stick. The aircraft starts turning in ever decreasing circles. The aircraft altitude is held constant. The quality of the manoeuvre is assessed by the lime linearity of 'g' profile. A rate of about 0.5-0.8, g/s is desirable. Once the maximum g is reached, the manoeuvre is complete.

The roller coaster manoeuvres were performed at 10,000 ft. and 20,000 ft., and wind up turn manoeuvres were conducted at 20,000 ft. and at Mach 0.6, 0.7 and 0.8. The slow down test was conducted at 10,000 ft. and at Mach 0.4 The aircraft was instrumented for measuring tri-axial acceleration very near C.G, Euler angles, angular rates, angle of attack and side slip, control surface positions and air data. The engine thrust was computed from measurements of indicated air speed, temperature and engine r.p.m. from thrust calibration curves. These data were supplied by flight test agencies to us for further data analysis.

### Aircraft Performance Estimation

The manoeuvres described above result into aircraft responses which are analysed using the procedure of system parameter estimation. Kinematic Consistency checks on the compatibility of these responses with the kinematics (dynamics) of the vehicles are performed. These checks reveal inconsistencies in the data like scale factor errors, biases etc., if any. At this stage no aerodynamic derivatives are included in the mathematical models. Once these errors are fixed, the data are then used in parameter estimation software along with appropriate mathematical models which include various important aerodynamic derivatives. The estimation method used for this purpose is described next.

#### a) Maximum Likelihood Method:

In general a dynamical system can be represented as follows:

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}, \mathbf{P}) \quad ; \quad \mathbf{x}(0) \text{ initial conditions}$$

$$\mathbf{y}(t) = \mathbf{h}(\mathbf{x}(t), \mathbf{u}, \mathbf{P}) \quad ;$$

$$\mathbf{z}(\mathbf{k}) = \mathbf{y}(\mathbf{k}) + \mathbf{v}(\mathbf{k}) \quad ; \quad \mathbf{k}=1,2,\dots,N$$

Using N sampled values of input-output time histories, the maximum likelihood problem can be formulated in a probabilistic manner by defining the likelihood function as the conditional probability density function of the measurements  $\mathbf{z}(\mathbf{k})$  given  $\mathbf{P}$  and  $\mathbf{R}$ . Here  $\mathbf{R}$  is the measurement noise covariance matrix and  $\mathbf{P}$  is the parameter vector. The likelihood can be maximised by minimising the negative log-likelihood function [3]:

$$J = \frac{1}{2} \sum_{k=1}^{N} [\mathbf{z}(\mathbf{k}) - \mathbf{y}(\mathbf{k})]^T \mathbf{R}^{-1} [\mathbf{z}(\mathbf{k}) - \mathbf{y}(\mathbf{k})] + \frac{N}{2} \ln |\mathbf{R}|$$

The MLE software performs this minimisation and yields the parameter vector P, initial conditions of states, generates the math model response and yields the Cramer-Rao bounds. The convergence of the determinant of R and cost function J evaluated at P are used for stopping the iterative procedure. The goodness of the fit is judged based on the Cramer-Rao bounds. The software can handle very long time history data and twelve-state model with as many as eight observables, It can be used to estimate as many as thirty parameters [8].

b) Modelling of the dynamic manoeuvres:  
Roller Coaster /Slow Down:

The performance parameters are estimated using the decoupled longitudinal equations of motion in the body-axis. The state and observation models are given below:

State Model:

$$\begin{aligned} \dot{u} &= \bar{q} S/m \{C_{x_0} + C_{x_u} u/V_0 + C_{x_w} \dot{w}/V_0\} \\ &\quad -q\omega - g\sin\theta + Fe/m \cos\sigma_T + b_u \\ &\quad \text{where } u(0) = u_0 \\ \dot{w} &= \bar{q} S/m \{C_{z_0} + C_{z_u} u/V_0 + C_{z_w} w/V_0 \\ &\quad + C_{z_{\dot{w}}} \dot{w}\} + qu + g\cos\theta - Fe/m \sin\sigma_T + b_w \\ &\quad \text{where } w(0) = w_0 \\ \dot{\theta} &= q + \dot{\theta}_0 \quad \text{where } \theta(0) = \theta_0 \\ \dot{q} &= \bar{q} S \bar{c} / I_y \cdot \{C_{m_0} + C_{m_u} u/V_0 \\ &\quad + C_{m_w} w/V_0 + C_{m_q} q \bar{c} / 2V_0 + C_{m_{\dot{w}}} \dot{w}\} \\ &\quad + Fe \{I_{xz} \sin\sigma_T + I_{yz} \cos\sigma_T\} / I_y \\ &\quad \text{where } q(0) = q_0 \end{aligned}$$

Observation model

$$\begin{aligned} V_m &= (u_n^2 + w_n^2)^{1/2} \\ \alpha_m &= \tan^{-1} \left( \frac{w_n}{u_n} \right) \\ \theta_m &= \theta \\ q_m &= q \end{aligned}$$

$$\begin{aligned} a_{u_n} &= q \bar{S}/m \{C_{x_0} + C_{x_u} u/V_0 + C_{x_w} w/V_0\} + Fe/m \cos\sigma_T \\ a_{w_n} &= q \bar{S}/m \{C_{z_0} + C_{z_u} u/V_0 + C_{z_w} w/V_0\} - Fe/m \sin\sigma_T \end{aligned}$$

with additional equations for measurements at nose boom

$$\begin{aligned} u_n &= u + q z_{\alpha} \\ w_n &= w - \alpha x_n \\ v &= (u_0^2 + w_0^2)^{1/2} \end{aligned}$$

The above mathematical model is used to generate the performance parameters from the measure responses for the roller coaster and slowdown manoeuver.

Windup Turn:

Since longitudinal and lateral dynamics are coupled in the windup turn manoeuver, the state for this manoeuver has coupled terms in the state equations as follows:

State Model:

$$\begin{aligned} \dot{u} &= \bar{q} S/m \{C_{x_0} + C_{x_u} u/V_0 + C_{x_w} w/V_0\} \\ &\quad -q\omega - g\sin\theta + Fe/m \cos\sigma_T + r v + b_u \\ \dot{w} &= \bar{q} S/m \{C_{z_0} + C_{z_u} u/V_0 + C_{z_w} w/V_0 \\ &\quad + C_{z_{\dot{w}}} \dot{w}\} + qu + g\cos\theta \\ &\quad - Fe/m \cos\sigma_T - p v + b_w \\ \dot{\theta} &= q \cos\phi - r \sin\phi \\ \dot{q} &= \bar{q} S \bar{c} \{C_{m_0} + C_{m_u} u/V_0 + C_{m_w} w/V_0 \\ &\quad + C_{m_q} q \bar{c} / V_0 + C_{m_{\dot{w}}} \dot{w}\} / I_y + Fe/I_y \\ &\quad \{I_{xz} \sin\sigma_T + I_{yz} \cos\sigma_T\} \\ &\quad + (I_z - I_x) / I_y p r + (r^2 - p^2) I_{xz} / I_y \end{aligned}$$

The lateral variables appearing in the above equations were used as **pseudo control inputs** in the estimation procedure.

**Observation Model:** It is the same as the model used for the roller coaster manoeuvre analysis.

### c) Performance Estimation

The drag polar of an aircraft can be determined from dynamic manoeuvres through a truncated Taylor series summation of the body axis components of lift and drag contributions due to various dynamic components:

$$C_x = C_{x_0} + C_{x_u} u/u_0 + C_{x_\alpha} \alpha$$

$$C_z = C_{z_0} + C_{z_u} u/u_0 + C_{z_\alpha} \alpha + C_{z_\delta} \delta$$

These body axis results can be converted to stability axis data using identities:

$$C_L = C_x \sin \alpha - C_z \cos \alpha$$

$$C_D = -C_x \cos \alpha - C_z \sin \alpha$$

From the static balance considerations:

$$W = L = q S C_{Ltrim}; D = \bar{q} S C_{Dtrim};$$

$$T - D - W \sin \gamma = 0$$

The lift and drag components are found by summing the corresponding  $C_x, C_y, C_z$  and  $C_{x_u}, C_{y_u}, C_{z_u}$  derivatives, estimated using the **decoupled longitudinal** modelling for roller coaster and slow down manoeuvre and the coupled equations of motion for the **windup** turn manoeuvre.

### Discussion of Results

Various roller coaster manoeuvres were analysed as dynamic manoeuvres by using kinematic consistency checks and parameter estimation method. These manoeuvres cover an angle of attack range from 0 deg (near zero g) to about 10 deg (at 2 g). The manoeuvres were performed reasonably well and the drag polars show good agreement with reference data. The fig. 1 shows the match between flight trajectory and MLE predicted response for a typical roller coaster manoeuvre. Fig 2 shows lift vs drag, lift vs angle of attack and drag vs angle of attack curves for a typical

roller coaster manoeuvre. Nine roller coaster manoeuvres were analysed and the collective drag polar is shown in fig 3. In each case the MLE estimates are compared with the reference values. It can be seen that the drag polar compares reasonably well with the reference values.

The windup turn manoeuvres were performed in a coupled mode and analysed accordingly. The lift and drag data for angle of attack range of 9 to 20 deg have been obtained from these manoeuvres. Fig 4 shows time history match for a typical wind up turn manoeuvre. The match is satisfactory except for the forward acceleration which is due to the poor resolution of the acceleration measurement. Fig 5 shows the lift, drag vs angle of attack and drag polar for a typical manoeuvre. Fig 6 shows the drag polar obtained by combining the estimation results from seven windup turn manoeuvres. The comparison with the reference values in this case indicate that the MLE estimates generally underestimate drag or overestimate lift by a small but consistent amount. The discrepancy could be due to the difference in the flight test configuration as compared to the reference configuration. Further it is not known authentically that the reference values are from flight test data.

Results from slow down manoeuvres have been obtained and a typical time history match is in Fig. 7. Fig. 8 shows the lift, drag vs angle of attack values compared with the reference values for a typical manoeuvre and Fig. 9 shows the drag polar obtained from the analysis of seven slow down manoeuvres. Fig. 10 gives the complete drag polar data for a total of 240 points. These points have been obtained by including points from all successfully analysed slow down, roller coaster and windup turn manoeuvre. These correspond to different speeds, altitudes, angle of attack range and load factors. Despite being from different manoeuvres, these points regress into a fairly small band of drag polar as shown in fig. 10. This exercise of flight test data has been very comprehensive and interesting from modelling point of view. It has been thus demonstrated that aircraft drag polars can be estimated with reasonably good accuracy from flight data generated by specially performed dynamic manoeuvres and by using parameter estimation methods. The experience with the present exercise has shown that there is some scope for refinement in measurements at instrumentation level. Within the given domain of accuracy and data acquisition, the results of performance estimation have been largely satisfactory. Further study and refinements are in progress.

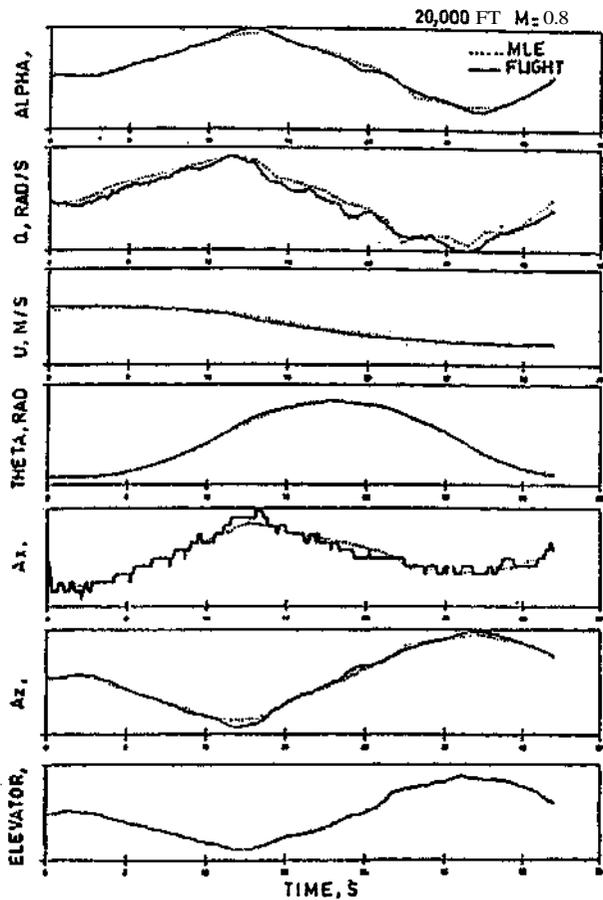


Fig. 1 Time History match for Roller Coaster Manoeuvre

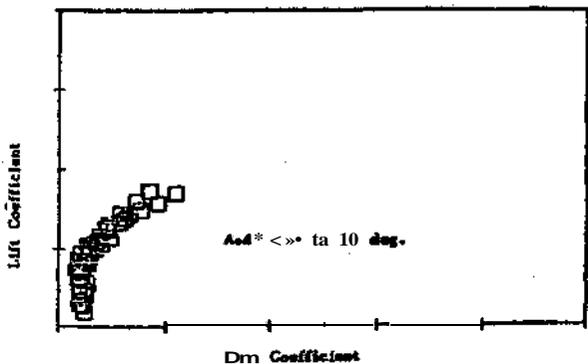


Fig. 3 Drag Polar from Roller Coaster Manoeuvres

PERFORMANCE IN A ROLLER COASTER MANOEUVRE, FLIGHT

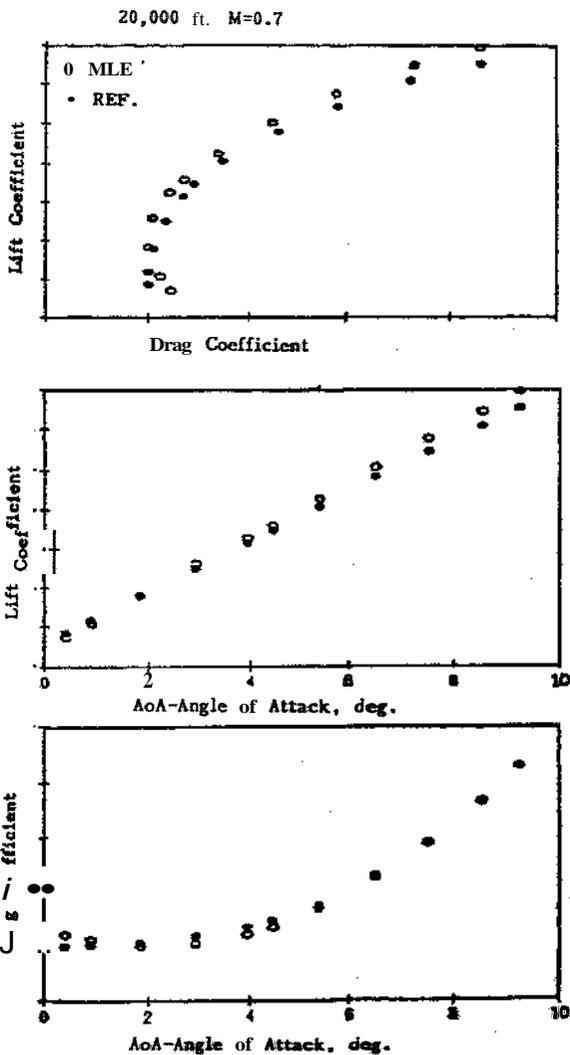


Fig. 2 Performance in a Roller Coaster Manoeuvre

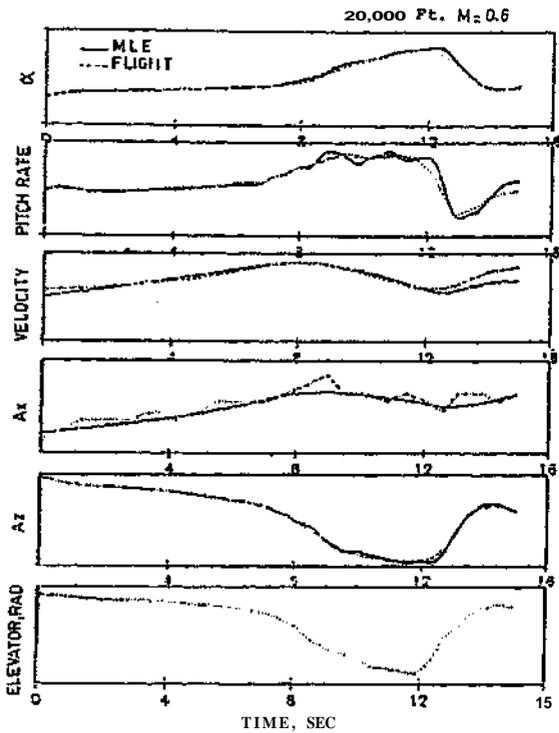


Fig. 4 Time History match for Windup Turn Manoeuvre

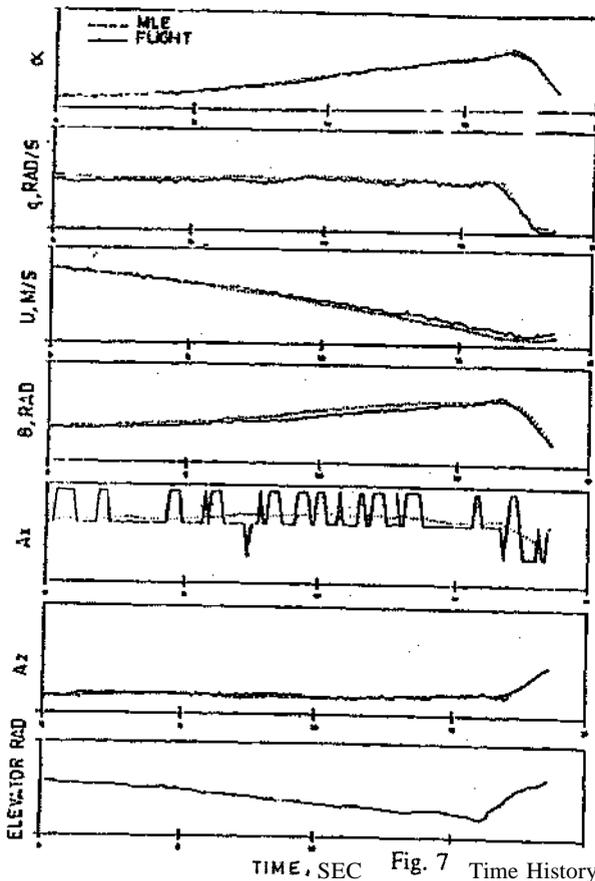


Fig. 7 Time History Match for Slowdown Manoeuvre

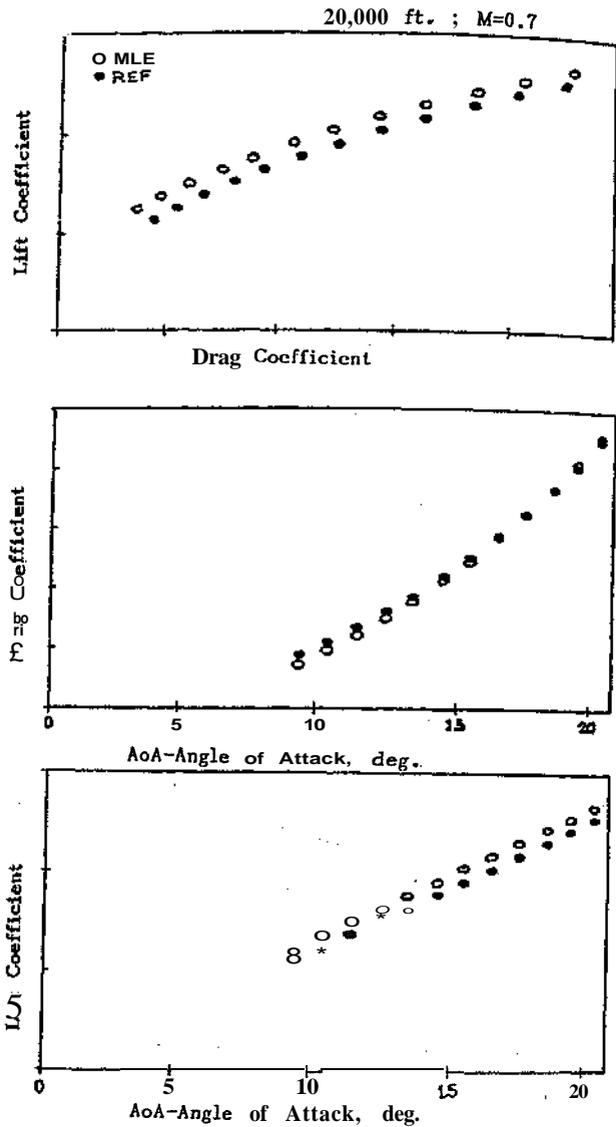


Fig. 5 Performance in a Windup Turn Manoeuvre

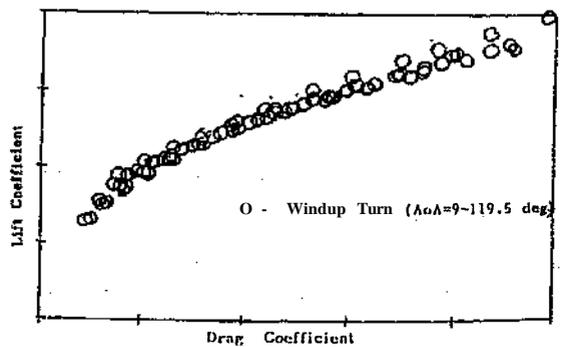


Fig. 6 Drag Polar from Windup Turn Manoeuvres

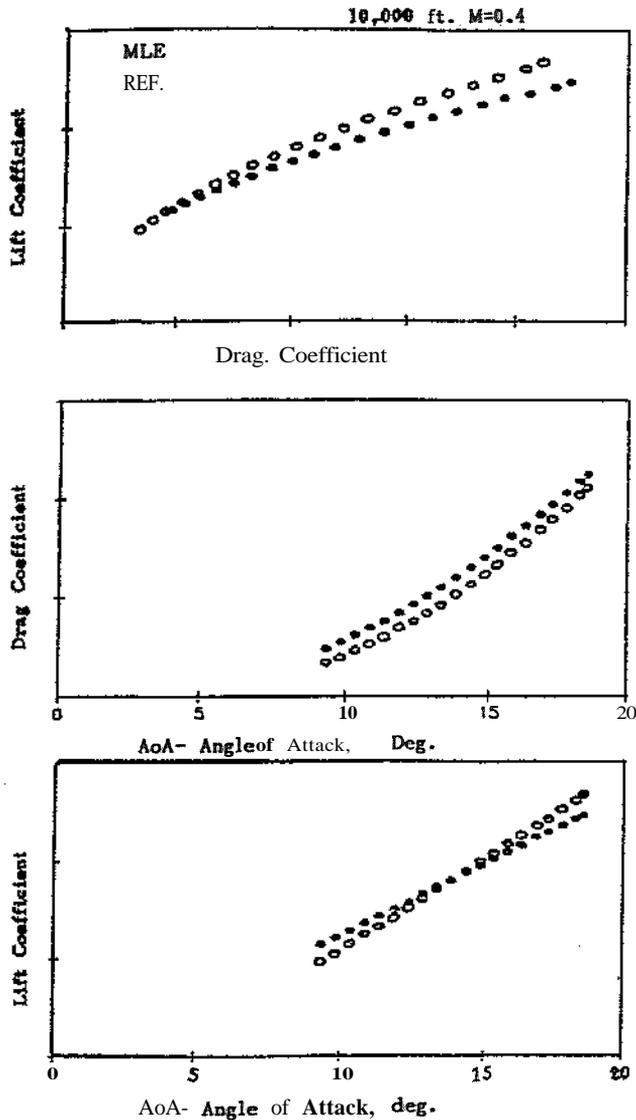


Fig. 8 Performance in a Slowdown Manoeuvre

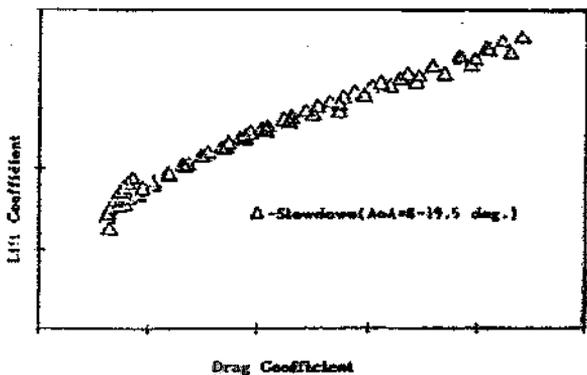


Fig. 9 Drag Polar from Slowdown Manoeuvres

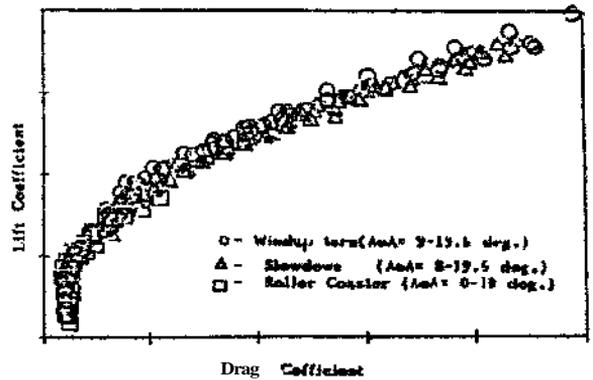


Fig. 10 Drag Polar of the Test Aircraft from Flight Tests.

**Conclusion**

In this paper results of performance estimation from dynamic manoeuvres of an aircraft have been presented. Planned flight test manoeuvres have yielded performance data that agree reasonably well with the values from other reference sources. The technique of maximum likelihood has been successfully used to estimate parameters of mathematical models fitted to the real flight data from extensive and comprehensive experiments conducted in the country.

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