Aircraft Turn Maneuver using Bifurcation Analysis and Nonlinear Dynamic Inversion Control Technique

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Abstract— **This paper demonstrates the application of Bifurcation analysis and continuation technique based methodology and Nonlinear Dynamic Inversion (NDI) control technique for designing aircraft maneuvers. A turning maneuver to the minimum sustained turn radius flight condition for F-18/ HARV aircraft model has been chosen to exemplify the proposed approach. A constrained bifurcation analysis based procedure is first carried out to construct the turn maneuver. The information available from bifurcation analysis is next used to specify the reference inputs for the NDI controller to switch the aircraft between the desired trim points. Closed-loop simulation results are also presented to show the effectiveness of the proposed methodology for designing aircraft maneuvers.**

I. INTRODUCTION

Aircraft flight typically involves switching of the aircraft trajectory between different operating points that lie within the restricted flight envelope of aircraft. The nature of initial and final states specifies the kind of maneuver and the ease with which an aircraft can be switched between these states signifies its maneuverability. Different kinds of minimum time or time-optimal maneuvers have been designed in the literature [1], [2], [3]. To enhance their agility, fighter aircraft are generally designed to be open loop unstable. Flying such an unstable platform requires a controller to stabilize the otherwise unstable configuration. The general approach to controller design followed in the industry is to linearize the nonlinear model of aircraft around various operating points and then use gain scheduling. However, lately there has also been a surge in interest in developing nonlinear controllers for attempting aircraft maneuvers. Control algorithms based on Nonlinear Dynamic Inversion (NDI)[4], Neural Network [5] and Sliding Mode Control (SMC) [6] techniques have been attempted with success for nonlinear aircraft models. The key idea behind design of controllers for aircraft nonlinear motions or maneuvers has been to switch an aircraft state from one to another using appropriate automatic control command generation.

Control algorithms based on NDI have been tried extensively in the literature. Enns et al. [7] used NDI to simulate the Herbst maneuver on a super-maneuverable aircraft model. NDI based controller was also utilized by Komduur et al. [8] to track an optimal trajectory for a Cobra-like maneuver for F-18/HARV mathematical model. Raghavendra et al. [9] applied NDI controller to show successful spin recovery of F-18/HARV aircraft model from a flat oscillatory spin state. Robust flight control systems based on nonlinear dynamic inversion technique have also been demonstrated [10].

Bifurcation analysis and continuation technique based methodology is a powerful tool for investigating the global dynamics and for computing steady states of nonlinear aircraft models with highly nonlinear aerodynamic characteristics [11], [12]. This technique has been used to analyze the nonlinear flight dynamics of both open-loop [13] and closed-loop[14] aircraft models. Bifurcation analysis and continuation technique has also been used to compute 'Attainable Equilibrium Sets' that are helpful in assessing the performance and agility of different aircraft models [15].

In this paper, we construct and simulate a turning maneuver for a six degrees-of-freedom F-18/HARV nonlinear airplane model using bifurcation analysis and continuation technique based methodology and an NDI controller. The organisation of paper is as follows. In section II, mathematical model of the aircraft used in present work is described. Construction of maneuver is discussed in Section III, followed by the description of NDI controller in Section IV. Section V presents the results obtained through the proposed technique, and Section VI concludes the paper.

II. MATHEMATICAL MODEL

Mathematical model of F-18/HARV is used for illustrating the technique. A complete 8th order nonlinear model for rigid body aircraft motion is used for the simulation. The geometrical parameters, mass and inertia characteristics, and aerodynamic database used for F-18/HARV are same as that used in Ref. 9. All the numerical simulation results presented in this paper are computed in MATLAB SIMULINK environment. The bifurcation analysis results shown in this paper have been obtained using AUTO 2000 continuation and bifurcation software [16].

III. MANEUVER DESCRIPTION AND COMPUTATION OF STEADY STATES

Here we consider a turning maneuver to minimum sustained turn radius flight condition. This maneuver involves transitioning the aircraft from a straight and level flight condition to the minimum sustained turn radius flight at constant speed. The maneuver is constructed as a two-point boundary value problem wherin the initial and final trim states of the maneuver are specified. To extract the trim states for the maneuver, Extended Bifurcation Analysis

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(EBA) method [17] is used. EBA method is a two step procedure that involves solving the equation of rigid body flight dynamics along with the relevant constraint equations in the first step. Thus,

$$\dot{\underline{x}} = \underline{f}(\underline{x}, u, \underline{p_1}, \underline{p_2}) = 0 \tag{1}$$

$$\underline{g}(\underline{x}) = 0 \tag{2}$$

In (1) and (2) , $\underline{x} = [V, \alpha, \beta, p, q, r, \phi, \theta]^T \in X \subset \Re^8$ is an eighth-dimensional vector of state variables with components defined as: V is the velocity of the airplane aligned along the flight path, α is the angle-of-attack, β is the sideslip angle; p, q, r are the body axis roll, pitch and yaw rates respectively, ϕ and θ refer to the body axis roll and pitch angle respectively. $u = [\delta_e] \in U \subset \Re$ is the principal continuation parameter : δ_e being the elevator angle. Vectors $\underline{p_1}$ and $\underline{p_2}$ denote free and fixed control parameters respectively. The vector function $\underline{g}(\underline{x})$ represents an (m-1-n($\underline{p_2}$)) -dimensional vector of constraints, where $n(\underline{p_2})$ is the number of fixed control parameters and m is the total number of control parameters available. In the second step of EBA, parameter schedules for the freed parameters, as obtained from the first step, are used to find the bifurcation diagram for the model under specified constraints. Thus, the second step of EBA solves:

$$\dot{\underline{x}} = \underline{f}(\underline{x}, u, \underline{p_1}(u), \underline{p_2}) = 0 \tag{3}$$

The minimum sustained turn radius that can be achieved by an aircraft is restricted by its aerodynamic, structural and propulsive limits. Reference 18 describes a method to find the maximum sustained turn rate of an aircraft using EBA method. Here, we use the same procedure to compute the minimum sustained turn radius flight states. Therefore, we solve for three different set of constraint equations together with the aircraft flight dynamic equations to compute the constrained trim states of the aircraft. These constraints are presented in Table 1. The symbols δ_a and δ_r in Table 1 pertain to aileron and rudder angles respectively, γ represents the flight path angle, η indicates the engine thrust as a fraction of maximum available engine thrust, n is the normal load factor and α_{stall} corresponds to the stalling angle-of-attack of the airplane.

TABLE I
SUMMARY OF CONSTRAINTS

Branch	Constraints($\underline{g}(\underline{x}) = 0$)	Free Parameters	Fixed Parameters
A	$\gamma = 0, \beta = 0$	δ_a, δ_r	$\eta = 1.0$
B	$\gamma = 0, \beta = 0,$ $\alpha = 0.62832 (= \alpha_{stall})$	δ_a, δ_r, η	-
C	$\gamma = 0, \beta = 0,$ $\phi = 1.3845 (n = 5.4)$	δ_a, δ_r, η	-

Figure 1 shows the plot of throttle schedules against elevator deflection as obtained from the first step of EBA. All branches give solution for level coordinated turn, but throttle parameter $\eta > 1$ on branches B and C. Hence, level coordinated turns are physically unrealizable on solution branches B and C. Sustained level turns are possible only on branch A. Figure 2 shows the plot of turn radius with Mach no. as computed from the second step of EBA. It can be noticed from Fig. 2 that minimum sustained turn radius flight condition occurs at point 'S' on branch A which is an open-loop stable trim point lying close to a bifurcation point.

The minimum radius turn states computed using EBA method are:

$$\mathbf{x_f} = [269.89, 0.37243, 0, 0.03289, 0.10726, -0.0842, -0.90534, 0.2367]^T \tag{4}$$

The initial steady, level, symmetric flight trim states can also be computed using EBA method by solving (1) with the following constraints :

$$\phi = 0, \gamma = 0, \beta = 0 \tag{5}$$

The result of such an analysis provides various level, symmetric trim conditions possible at a particular altitude. The initial trim condition can then be selected from the computed solution set. The initial trim states and control vector components for the turn maneuver calculated using EBA procedure are given as :

$$\mathbf{x_i} = [269.89, 0.1888, 0, 0, 0, 0, 0, 0.1888]^T \tag{6}$$

$$\eta = 0.3863, \delta_e = -0.0168, \delta_a = 0, \delta_r = 0 \tag{7}$$

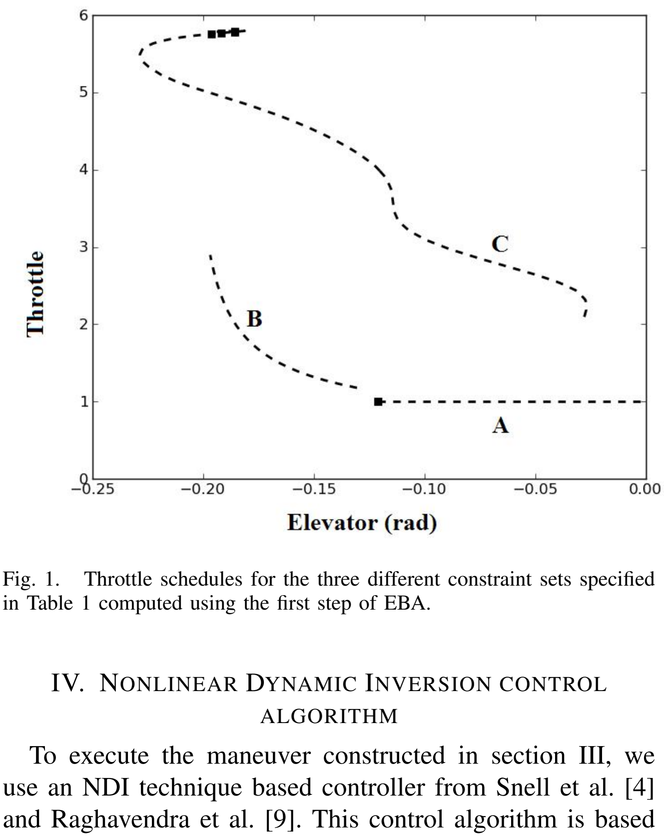

Fig. 1. Throttle schedules for the three different constraint sets specified in Table 1 computed using the first step of EBA.

IV. NONLINEAR DYNAMIC INVERSION CONTROL ALGORITHM

To execute the maneuver constructed in section III, we use an NDI technique based controller from Snell et al. [4] and Raghavendra et al. [9]. This control algorithm is based on the time scale separation of aircraft dynamics into slow and fast modes and is implemented as an inner/outer loop design. The angular rates p, q, and r denote the fast variables

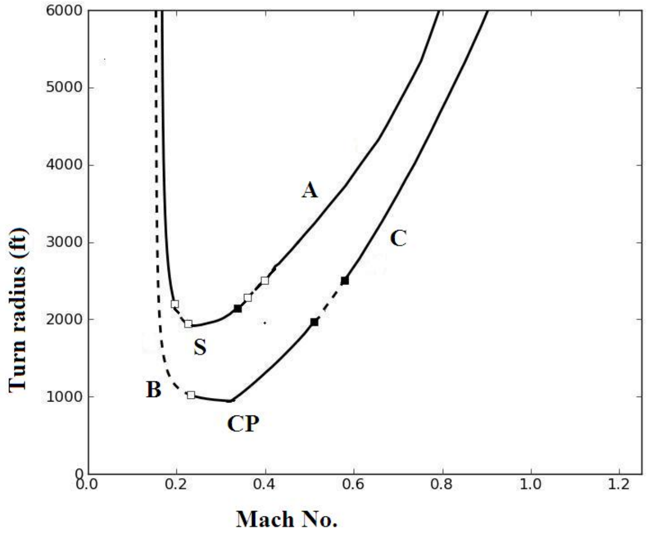

Fig. 2. Turn radius variation with Mach No.; solid line: stable point, dotted line: unstable point, empty square : transcritical or pitchfork bifurcation, filled square: Hopf bifurcation.

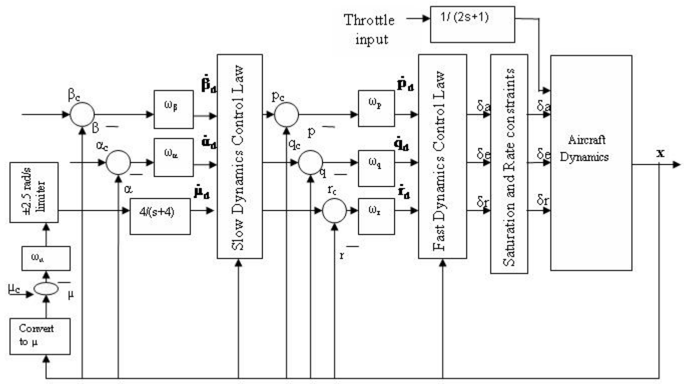

Fig. 3. Block diagram for NDI controller.

while the angles α, β, and μ correspond to the slow variables. In the block diagram shown in Fig.3, the outer loop inverts the dynamics of slow variables to obtain the commanded values of fast variables, p_c, q_c and r_c. The desired dynamics for the fast variables are specified by passing p_c, q_c, and r_c through respective bandwidth blocks. The inner loop then inverts the dynamics of fast variables to obtain the required control inputs for achieving the desired fast dynamics. The bandwidths, ω_p, ω_q and ω_r, for fast states are all set to 10 rad/s. The bandwidths, ω_α and ω_β, of slow states are each set at 2 rad/s while ω_μ is fixed at 1.5 rad/s. It is further assumed that all the states of the system are available for feedback. It will be shown through our results in the next section that this algorithm leads to an oscillatory response for the turn maneuver. This oscillatory response is found to be caused by the rate saturation of rudder actuator. To avoid this oscillatory behaviour of aircraft variables, we try to reduce the bandwidths of both the inner and outer loops by trial and error while maintaining the ratio between the bandwidths of two loops. The modified values of bandwidths used are: $\omega_p = \omega_q = \omega_r = 7.5$ rad/s, $\omega_\alpha = \omega_\beta = 1.5$ rad/s and ω_μ = 1.0 rad/s. As shown through the results of next section, the turn maneuver can be successfully attempted with these modified values of bandwidths.

V. RESULTS AND DISCUSSIONS

Closed loop simulation results for the turn maneuver are presented in this section. To initiate the maneuver, the aircraft is assumed to be flying open loop in a steady, straight and level, symmetric flight condition for about 10 seconds. NDI controller is then activated after 10 seconds to simulate the maneuver. It can be observed from Fig. 4 that aircraft has not settled down into the minimum sustained turn radius trim condition, but instead shows an oscillatory response in sideslip and roll angle variables. To avoid this oscillatory behaviour, we modify the bandwidths of slow and fast variables by trial and error method as discussed in section IV. The results for the turning maneuver with modified bandwidths are presented in Figs. 6 and 7. It can be seen from the plots of Fig. 6 that the aircraft stabilizes into the desired final state within 10 seconds of the initiation of controller action, although mach no. and pitch angle (θ) settles down to the final steady state values a little later. The NDI generated control commands are shown in Figs. 7 (c) and 7 (d). The initial sense of control commands is to give a left rudder along with a left aileron to maintain a coordinated turn to the left. A simultaneous nose-up elevator increases the angle-of-attack to generate extra lift required to achieve a level turn at a constant velocity. The three-dimensional flight path trajectory of the aircraft for turning maneuver is presented in Fig.5.

VI. CONCLUSIONS

This study has illustrated the use of bifurcation analysis and continuation technique based methodology along with NDI control technique for designing and executing aircraft maneuvers. Bifurcation analysis is first used to extract the initial and final trim state vectors for constructing a turning maneuver to minimum sustained turn radius flight condition. Closed-loop simulation of the turn maneuver is then performed using bifurcation analysis results as set-points for the NDI controller. The bandwidths for slow and fast variables of NDI controller are fixed by trial and error to alleviate the actuator rate saturation problems. The simulation results for the turn maneuver highlight the effectiveness of the proposed approach.

VII. APPENDIX

TABLE II

ACTUATOR CONSTRAINTS

Control surface	Position limit (deg.)	Rate limit (deg./s)
Elevator	(-25,10)	$\pm$ 40
Aileron	(-35,35)	$\pm$100
Rudder	(-30,30)	$\pm$82

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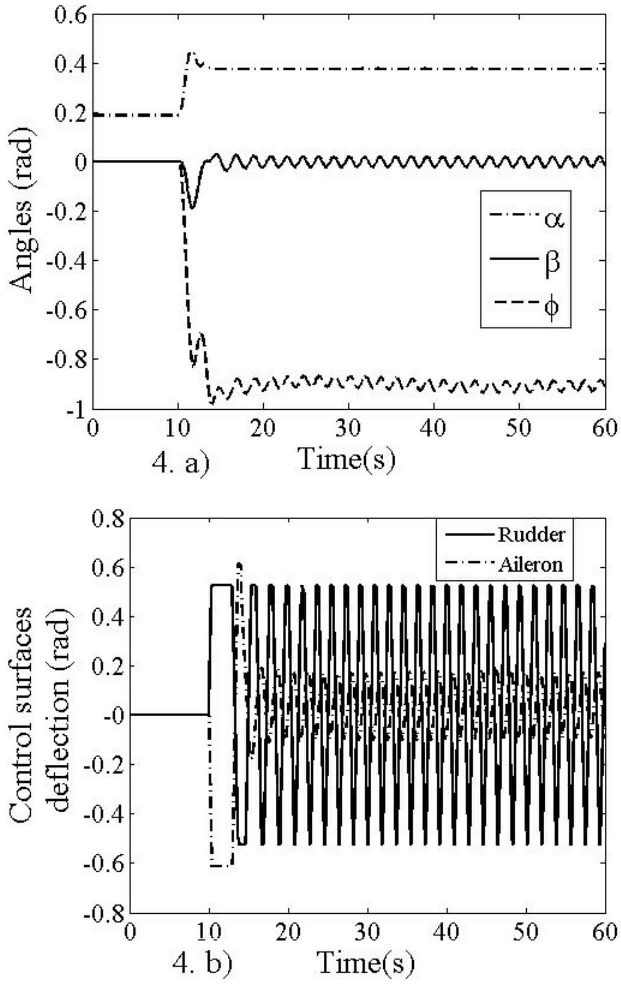

Fig. 4. (a)Time response of aircraft parameters (b) Control input history for the turning maneuver using NDI algorithm with bandwidths: $\omega_p = \omega_q = \omega_r$ = 10 rad/s, $\omega_\alpha = \omega_\beta$ = 2 rad/s and ω_μ = 1.5 rad/s.

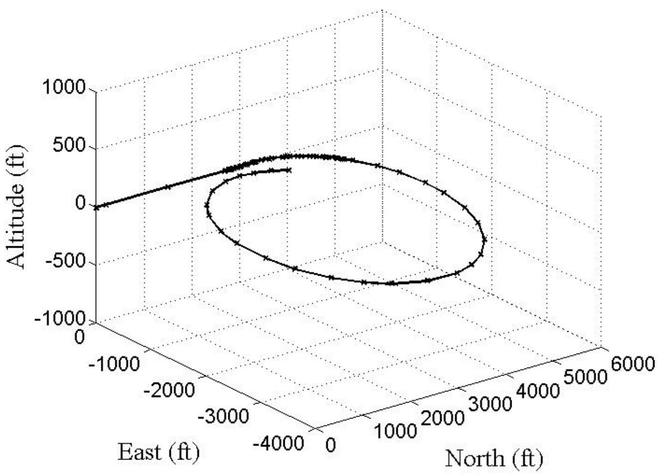

Fig. 5. Aircraft flight path trajectory for turning maneuver using NDI controller with modified bandwidths.

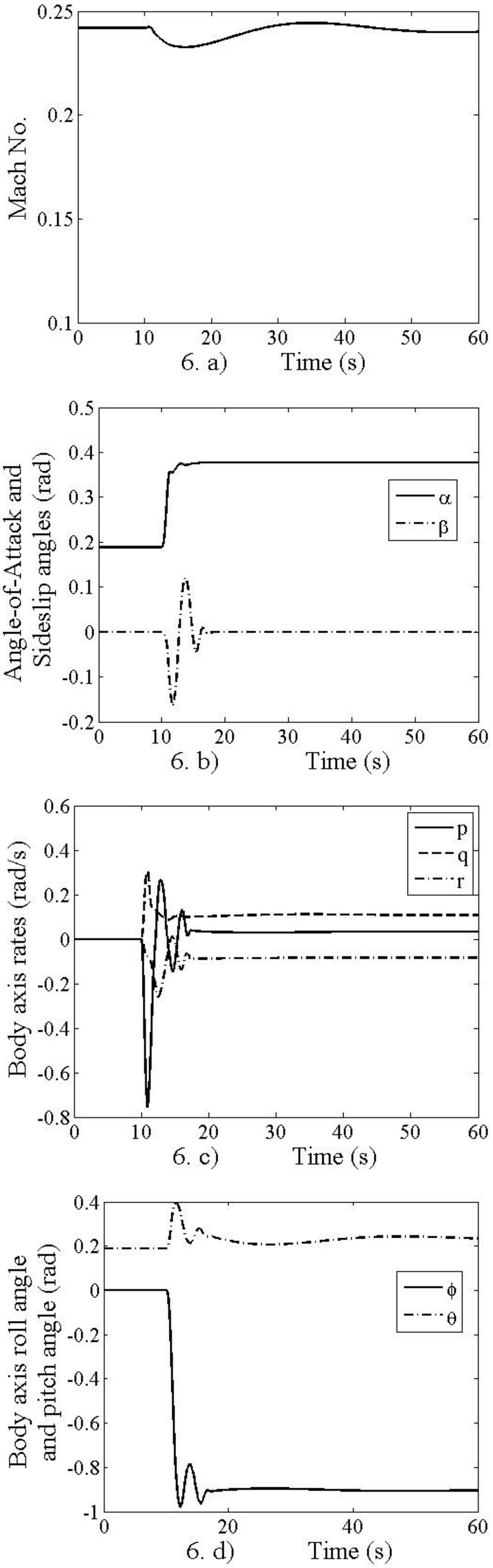

Fig. 6. Time history of aircraft parameters for turning maneuver using NDI controller with modified bandwidths: $\omega_p = \omega_q = \omega_r = 7.5$ rad/s, ω_α = ω_β = 1.5 rad/s and ω_μ = 1.0 rad/s.

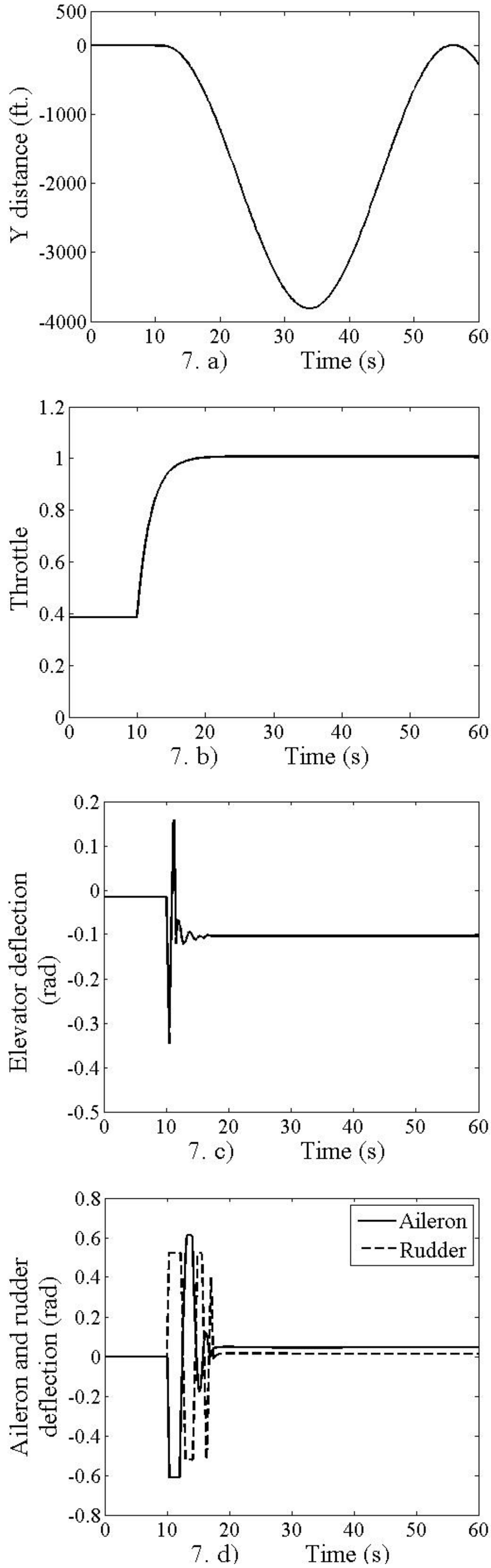

Fig. 7. a) Plot of Y distance, (b) Throttle input, (c) and (d) Time history of NDI control commands for turning maneuver with modified bandwidths: $\omega_p = \omega_q = \omega_r = 7.5$ rad/s, ω_α = ω_β = 1.5 rad/s and ω_μ = 1.0 rad/s.