THE EFFECT OF PERIODIC BALLONET JET EXHAUST ON THE STABILITY OF A TETHERED AEROSTAT

S.M. Kannan
National Aeronautical Laboratory
Bangalore 560 017, India

G.N.V. Rao
Indian Institute of Science
Bangalore 560 012, India

Abstract

Tethered aerostats operating under wide range of altitudes and atmospheric pressure conditions employ ballonets within the hull filled with air. These ballonets exhaust the air into the atmosphere whenever necessary to maintain the difference between the atmospheric pressure and the internal pressure within a desired range. Assuming that the air exhaust is made periodic, studies are made on the effect of the periodic jet force and the resulting pitching moment on the response of the aerostat system. In the first phase, the stability of the system is studied by analysing the eigenvalues and eigenvectors. The result shows that the aerostat system is unstable in the windspeed range of 0-15 fps and stable in the windspeed range of 15-100 fps. When the system is subjected to a suitable periodic forcing from the exhaust jet, it is observed that it is not only possible to extend the stability boundary of 15 fps down to typically 5 fps, but much more interestingly, there exist forcing periods of the exhaust jet which render the system completely stationary.

Nomenclature

A
matrix of coefficients of the system dynamical eqs. (12)

B
total buoyant force of the aerostat

c
axial ballonet constraint term used in eq. 11

dx /d
axial ballonet constraint term used in eq. 11

E
matrix of coefficients of the system dynamical eqs. (12)

F
forcing function vector

g
gravitational acceleration

m
moments of inertia about the y and z axes, respectively

K
spring constant defined by eq. 10

m
mass of the aerostat excluding the hull's internal air and gas.

M
equivalent mass term defined by eq. 9

m , m
masses of the hull's internal air and gas, respectively.

N
number of cycles for unforced ballonet slosh to damp to half amplitude

q
angular velocity about the y-axis

Q
Complex eigenvector term for q from eq. 13

t
time

T
tether cable tension

U
complex eigenvector term for \( \delta T \) from eq. 13

v , v
mass-centre velocities of \( m_a \) and \( m \), respectively

V
volume

V
Complex eigenvector term for \( v \) from eq. 13

W
weight of the aerostat system

W
velocity of the mass centre of \( m \) in the z-direction

X , Z
body-fixed wind axes originating at the mass centre of \( m \)

Y
body-fixed coordinate normal to x and z axes

X , Y , Z
forces on \( m \) in the \( x, y, z \) directions, respectively

\( \delta X_a \)
complex eigenvector term for \( \delta X_a \) from eq. 13

Greek symbols

\( \alpha \)
aerodynamic angle of attack

\( \gamma \)
perturbed cable angle

\( \Gamma \)
cable angle defined in Fig. 1

\( \hat{\gamma} \)
complex eigenvector term for \( \gamma \) from eq. 13

\( \delta ( ) \)
quantity perturbed from equilibrium

\( \Re \)
real part of a

\( \theta \)
aerostat pitch angle defined in Fig. 1

\( \hat{\theta} \)
complex eigenvector term for \( \theta \) from eq. 13
A tethered aerostat (Fig.1) designed to operate over a wide range of altitudes and varying atmospheric pressure is usually provided with what is called a 'ballonet' (Fig.2). The ballonet is simply another flexible bag inside the hull, which is continuously filled with the atmospheric air by means of a blower. This flexible bag communicates pressure to the gas in the hull. The internal pressure is typically kept in the range of 5-20 cm of water column above the atmospheric pressure. In order to maintain the rigidity of the hull, without exceeding the allowable loadings on the hull structure, when the pressure exceeds the set higher limit, a pressure valve in the ballonet opens and discharges the excess air into the atmosphere. Since the blower is kept running all the time, the discharge valve operates intermittently so as to maintain the internal pressure within the higher limit.

A theoretical investigation was undertaken to study the effect of the impulsive periodic action of the exhaust jet on the system response, together with the sloshing motions of the air mass in the ballonet described by Delaunay et al. The effect of variation of the cable tension along its length is not included in the present analyses.

Formulation of the Eigenvalue Problem

In the first phase of the study, the natural stability of the aerostat system is studied, without the action of the exhaust jet. The equations of motion of the system are given by the following eight simultaneous first order linear differential equations:

\[ X_{a} \ddot{u} + X_{w} \ddot{w} + \ddot{q} + (B - W_{o} + \frac{T_o \sin \theta}{\theta}) e + dT \cos \theta - T_o \sin \theta \gamma + (X_{u} + W_{o}/V_{a}) u - (X_{w} w) - (m_{a} \dot{z}_{ao} + m_{g} \dot{g}_{ao} - X_{q} - (m_{a} - m_{g} \ V_{a}/\gamma) V_{a} = 0 \]  

(eq. (1)

(Force equilibrium equation in the x direction)

\[ Z_{u} \ddot{u} + Z_{w} \ddot{w} + \ddot{z} + (W_{o}/\gamma) u + T_o \cos \theta + \sin \theta \delta T + T_o \cos \theta \gamma \]  

(eq. (2)

(Force equilibrium equation in the z direction)

\[ M_{u} \ddot{u} + M_{w} \ddot{w} + (M_{q} - (m_{a} \ddot{z} + m_{g} \ddot{g})) u + (Bz_{a} - m_{a} \ddot{z}_{ao} - m_{g} \ddot{g}_{ao}) g \]  

\[ (m_{a} - m_{g} \ V_{a}/\gamma) g \delta x + (x_{u} \dot{z}_{ao} - x_{w} \dot{g}_{ao} + z_{ao} - z_{g} - \beta_{ao} \dot{z}_{ao} = 0 \]  

\[ \begin{align*}
    X_{a} & = 0 \\
    q & = 0 \\
    \dot{\delta} & = 0 \\
    \dot{\epsilon} & = 0 \\
    \omega - U_{o} & = 0 \\
    \dot{x}_{u} & = 0 \\
    \dot{x}_{w} & = 0 \\
    \dot{z}_{ao} & = 0 \\
\end{align*} \]  

(eq. (4) to (7) are kinematic relations)

\[ \begin{align*}
    (m_{a} - m_{g} \ V_{a}/\gamma) g \delta x_{a} + (m_{e} - m_{g} \ V_{a}/\gamma) g \dot{\delta} \dot{x}_{a} + \dot{\delta} \dot{\delta} x_{a} = 0 \\
\end{align*} \]  

(eq. (8)

(dynamic equation of the air and gas relative to the hull)

where,

\[ \begin{align*}
    \dot{\delta} & = 0.2206 (K \dot{m})^{2} / \left[ (N_{a})^{2} \right] \frac{0.1217}{(\delta x)^{2}} \\
\end{align*} \]  

(eq. (9)

The unknown quantities \( \theta, \phi, \psi, T_{o}, \) and \( \dot{T}_{o} \) are determined from the longitudinal, vertical and pitching moment equilibrium equations. The above
set of 8 simultaneous equations, may be represented in the following matrix notation:

\[
\begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{v}_0 \\
\dot{x}_a \\
\dot{s}_T \\
\dot{v}
\end{bmatrix} = 0 \quad \text{(12)}
\]

where \([A]\) and \([E]\) are 8x8 matrices whose elements are the coefficients of the 8 equations. With the assumption of a harmonic solution,

\[
[Y] = \begin{bmatrix}
\dot{u} \\
\dot{w} \\
\dot{q} \\
\dot{v}_0 \\
\dot{x}_a \\
\dot{s}_T \\
\dot{v}
\end{bmatrix} = \begin{bmatrix}
\hat{\dot{u}} \\
\hat{\dot{w}} \\
\hat{\dot{q}} \\
\hat{\dot{v}}_0 \\
\hat{\dot{x}}_a \\
\hat{\dot{s}}_T \\
\hat{\dot{v}}
\end{bmatrix} e^{\sigma t} \quad \text{(13)}
\]

equation (12) leads to an eigenvalue problem. From equations (12) and (13), we obtain

\[
([A] - \sigma [E]) [Y] = 0 \quad \text{(14)}
\]

or, \([A] - \sigma [E] = 0 \quad \text{(15)}
\]

Pre-multiplying by \(A^{-1}\), we get

\[
[1 - \sigma A^{-1}E] = 0 \quad \text{(16)}
\]

The equations of motion (12) with the forcing inputs therefore take the final form:

\[
[A] [Y] - [E] [Y] + [FF] = [O] \quad \text{(18)}
\]

The forcing function vector thus becomes

\[
\begin{bmatrix}
0 \\
-9 \\
-100 \\
0 \\
0 \\
0 \\
0
\end{bmatrix} \quad \text{(17)}
\]

The equations of motion (12) and the derivatives of the state variables \(\dot{y}\) obtained as follows:

\[
\dot{Y}_1 = \Sigma A_{1j} Y_j - E_{13} \quad \dot{Y}_2 = \Sigma A_{2j} Y_j - E_{23} \quad \dot{Y}_3 = \Sigma A_{3j} Y_j - E_{31} \quad \dot{Y}_4 = \dot{Y}_3 - E_{32} \quad \dot{Y}_5 = -E_{35} \quad \dot{Y}_6 = FF(2) \quad \dot{Y}_7 = FF(3)/E_{33} \quad \dot{Y}_8 = Y_1 - E_{64} Y_4 / E_{68}
\]

where \(\Sigma\) stands for summation over \(j = 1, 3\).

These are a system of 8 first order linear differential equations, which are solved using Runge-Kutta-Gill's numerical integration method.

Calculations of the response were made as follows: The initial perturbations were taken to be zero and the periodic operation of the jet exhaust assumed. Calculations were performed for one small value of the forcing amplitude and another at 10 times this value to see the effect of the amplitude of the forcing function. It was noticed that the amplitude of the various state variables started increasing in most cases, although the rate of increase depended on the forcing period. It was observed that corresponding to the large forcing amplitude, there were two forcing periods at all wind speeds at which, there was practically no build-up of the state variables with time. The results thus obtained are shown.

System Response to Dynamical Inputs

The balloon air exhaust jet is assumed to operate periodically in a square wave pattern. The range of forcing periods studied is 5-100 sec. The exhaust jet velocity is assumed to be 100 fps and the jet area 0.35 sq.ft. corresponding to 8 inch dia. outlet. The jet is assumed to act downwards normal to the body axis of symmetry at a distance of 11 ft. from the C.G. of the system. The jet reaction force and the resulting pitching moment are obtained from the relations:

\[
\begin{align*}
FF(2) &= -\rho AV^2 = -9 \text{ lb.} \\
FF(3) &= -9 \times 11 \text{ ft. lb.} = -100 \text{ ft. lb.}
\end{align*}
\]

The forcing function vector thus becomes

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FF = \begin{bmatrix} 0 \\ -9 \\ -100 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{(17)}
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From these equations, the derivatives of the state variables \(\dot{y}\) obtained as follows:

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\dot{Y}_1 &= \Sigma A_{1j} Y_j - E_{13} \\
\dot{Y}_2 &= \Sigma A_{2j} Y_j - E_{23} \\
\dot{Y}_3 &= \Sigma A_{3j} Y_j - E_{31} \\
\dot{Y}_4 &= \dot{Y}_3 - E_{32} \\
\dot{Y}_5 &= \Sigma A_{5j} Y_j + FF(2) \\
\dot{Y}_6 &= \Sigma A_{5j} Y_j + FF(3) \\
\dot{Y}_7 &= Y_1 - E_{64} Y_4 \\
\dot{Y}_8 &= Y_1 - E_{64} Y_4 / E_{68}
\end{align*}
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where \(\Sigma\) stands for summation over \(j = 1, 3\).

These are a system of 8 first order linear differential equations, which are solved using Runge-Kutta-Gill's numerical integration method.

Calculations of the response were made as follows: The initial perturbations were taken to be zero and the periodic operation of the jet exhaust assumed. Calculations were performed for one small value of the forcing amplitude and another at 10 times this value to see the effect of the amplitude of the forcing function. It was noticed that the amplitude of the various state variables started increasing in most cases, although the rate of increase depended on the forcing period. It was observed that corresponding to the large forcing amplitude, there were two forcing periods at all wind speeds at which, there was practically no build-up of the state variables with time. The results thus obtained are shown.
in Figs. 4, 5 and 6. These calculations were made for two values of the amplitude of the forcing function.

**Discussions of the Results**

The most interesting observation from the above results is that there appears to be a specific period of the forcing function and its amplitude at all wind speeds at which the response parameters like perturbation velocities, pitch angle, pitch rate etc. remain practically zero. If the forcing amplitude is sufficiently large at a given wind speed, it appears to be possible to drive the system parameters to nearly zero magnitudes at small values of the forcing period, in addition to that at large forcing period. However, it seems relatively easy to drive the system amplitudes to nearly zero values at the second forcing period, which is larger than the first one. At the lowest wind speed of 10 fps, it is observed that the response amplitudes reach a minimum value at a forcing period of approximately 12 sec. This period is close to the natural short period mode of the aerostat (Fig. 4). As the wind speed is increased, two distinct forcing periods are observed (Figs. 5 and 6) at the higher amplitudes of the forcing function and the separation between these two periods increases with increasing wind speed. In fact, one could say that the two critical forcing periods nearly coincide at low wind speeds around 10 fps. However, if the amplitude of the forcing function is small, only one forcing period at which near-zero response amplitude is observed. The larger forcing period at which the system response is close to zero is a weak function of the amplitude of the forcing function. In Fig. 7, the forcing period for zero response has been plotted against wind speed for two amplitudes of the forcing function.

Also shown in Fig. 7 are the first and the third modes of oscillation of the system which correspond approximately with the inverted pendulum and the short period modes. One observes that while the inverted pendulum mode may have a bearing on the response amplitude of the system at low speeds, the critical forcing period for either suppressing or minimizing the amplitude response lies somewhere in between the two natural periods.

**Conclusions**

The basic system is unstable in the wind speed range of 0-15 fps and stable in the wind speed range of 15-100 fps. When the system is subjected to a suitable periodic forcing from the exhaust jet, it is not only possible to extend the stability boundary of 15 fps down to 5 fps, but much more interestingly, there exist forcing periods of the exhaust jet at which the system exhibits zero response. This indicates that a suitably tuned periodic jet exhaust can be employed as a dynamk vibration absorber and stabilise the system against all perturbation motions and render the tethered aerostat system practically stationary in the longitudinal mode. The limited studies reported in this paper consider only the symmetric motion of the aerostat. It is well known that the lateral oscillations of the aerostat, particularly of the inverted pendulum mode, are perhaps of even greater importance. Any extension of the present approach for stabilising the aerostat in its lateral mode should assume that the discharge of the ballonet air should also be made in the lateral direction.

**References**


Fig. 3 Variation of real part of stability roots with wind speed.

Fig. 4 Response at 5 minutes at wind speed=60 fps. vs. forcing period.

Fig. 5 Response at 5 minutes at wind speed=42.2 fps. vs. forcing period.

Fig. 6 Response at 5 minutes at wind speed=100 fps. vs. forcing period.

Fig. 7 Response at 5 minutes at wind speed=100 fps. vs. forcing period.

Fig. 8 Forcing period for zero response vs. windspeed.
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Nomenclature

A matrix of coefficients of the system dynamical eqs. (12)
B total buoyant force of the aerostat
damping coefficient defined in eq. 11
\( \Delta x_a/\Delta \theta \) axial ballonet constraint term used in eq. 11
E matrix of coefficients of the system dynamical eqs. (12)
FF forcing function vector
gravitational acceleration
moments of inertia about the y and z axes, respectively
K spring constant defined by eq. 10
m mass of the aerostat excluding the hull's Internal air and gas.
M equivalent mass term defined by eq. 9
m_a, m_g masses of the hull's Internal air and gas, respectively.

\( N \) number of cycles for unforced ballonet slosh to damp to half amplitude
\( \dot{q} \) angular velocity about the y-axis
\( \Delta \dot{q} \) Complex eigenvector term for \( q \) from eq. 13
t time
T tether cable tension
\( \Delta T \) complex eigenvector term for \( \Delta T \) from eq. 13
\( \Delta U \) complex eigenvector term for \( U \) from eq. 13
u velocity of the mass centre of \( m \) in the x-direction
v_0 mean wind speed
v_a, v_g mass-centre velocities of \( m_a \) and \( m_g \) respectively
\( V \) Volume
\( V_a \) Complex eigenvector term for \( v_a \) from eq. 13
\( W \_0 \) weight of the aerostat system
w velocity of the mass centre of \( m \) in the z-direction
\( \Delta W \) complex eigenvector term for \( w \) from eq. 13
\( x, z, y \) body-fixed wind axes originating at the mass centre of \( m \)
\( \gamma \) body-fixed co-ordinate normal to \( x \) and \( z \) axes
\( X, Y, Z \) forces on \( m \) in the \( X, Y, Z \) directions, respectively
\( \Delta x_a \) complex eigenvector term for \( \Delta x_a \) from eq. 13

Greek symbols

\( \alpha \) aerodynamic angle of attack
\( \gamma \) perturbed cable angle
\( \Gamma \) complex eigenvector term for \( \gamma \) from eq. 13
\( \delta \) Quantity perturbed from equilibrium
\( \eta \) real part of \( a \)
\( \beta \) aerostat pitch angle defined in Fig. 1
\( \Delta \beta \) complex eigenvector term for \( \beta \) from eq. 13

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