ESTIMATION OF NEUTRAL AND MANEUVER POINTS OF AIRCRAFT BY DYNAMIC MANEUVERS

S. Srinathkumar, Padma Madhuranath, and Girija Gopalratnam
National Aerospace Laboratories, BANGALORE 560017, INDIA

Abstract

A new flight test technique, based on aircraft parameter estimation methods, is proposed to simultaneously determine the neutral and maneuver point of aircraft. The new procedure is derived by relating the neutral point and maneuver point of an aircraft to key short period parameters $M_a$ and short period natural frequency $\omega_n^2$ respectively. The new flight test method results in substantial savings in flight test time compared to conventional methods. The method is more accurate since only inertial sensor data (pitch rate and normal acceleration) is used in the estimation procedure.

Introduction

The Neutral Point $N_0$ and Maneuver Point $N_m$ are important longitudinal stability parameters which critically determine the Aft CG limit of an aircraft. Since these parameters are a function of speed, angle of attack, external store configuration, control surface deployment (slats) etc., extensive flight tests are conducted to accurately determine these critical stability parameters. Existing methods based on steady-state trim flights turn out to be time consuming and are prone to error due to the results being dependent on air data and aircraft weight data. In this paper an alternative flight test methodology, based on dynamic maneuvers followed by modern aircraft parameter estimation analysis methodology, is proposed to determine $N_0$ and $N_m$ simultaneously. This results in substantial reduction in flight test time. Further the estimation of the stability parameters are independent of air data, mass or inertia data of the aircraft and depend only on the accuracy of CG position of the aircraft and the accuracy of inertial sensors (pitch rate and normal acceleration).

Definitions

The Neutral Point $N_0$ is defined as the CG position for which, in straight and level flight conditions (1g),

$$\frac{dC_m}{dC_l} = 0 \text{ or equivalently } \frac{d\delta_e}{dC_l} = 0$$

(1)

where $C_m$ is the moment coefficient, $C_l$ is the Lift Coefficient and $\delta_e$ is the elevator position. The distance between $N_0$ and actual CG position $(N_0 - \bar{x}_{CG})$ is called the static margin. $N_0$ and $\bar{x}_{CG}$ are defined as a percentage of an aircraft reference length, typically the mean aerodynamic chord (mac) denoted by $\bar{c}$.

The Maneuver Point, $N_m$, is defined as the CG position at which, under steady pull-up maneuvers, (in which the velocity and angle of attack are held constant)

$$\frac{dc_m}{dC_l} = 0 \text{ or equivalently } \frac{d\delta_e}{dn} = 0$$

(2)

where "$n$" is the load factor, defined as the ratio of Lift to Weight. $N_m$ is again defined as a percentage of $\bar{c}$. The distance $(N_m - \bar{x}_{CG})$ is called the maneuver margin. It should be pointed out that under accelerated flight condition additional stability accrues due to pitch rate damping and thus $N_m$ is invariably aft of $N_0$.

Conventional Flight Tests To Determine $N_0$ and $N_m$

Determination of $N_0$ by flight tests is usually done by measuring elevator angle for trim in steady flight, at a number of air speeds for different CG positions. For each reference $C_l$, and CG position the slope $\frac{d\delta_e}{dC_l}$ is computed. Then

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$N_0$ is determined graphically by noting the $CG$ position where $\frac{d\delta_r}{dC_L} = 0$.

Manipulation point $N_m$ is determined from flight tests by analysing data from pull-up maneuvers. The pilot sets up a shallow dive at the speed and power called for in the flight condition of interest. He then pulls back on the stick and attempts to hold a steady predetermined "$g$" (load factor) on his accelerometer. If he is skilful, stick position and accelerations are momentarily steady with the desired airspeed holding about constant. In this technique, the elevator position for trim is really being used as an indicator of $C_{m}$, and $n$ is, of course, an indicator of $C_L$, and so that the $CG$ position

where $\frac{d\delta_r}{dC_L} = 0$ implies $\frac{dC_{m}}{dC_L} = 0$.

This experiment is repeated for several load factors and different $CG$ positions. The slope $\frac{d\delta_r}{dn}$ for each $CG$ position is computed. Then $N_m$ is graphically determined by noting the $CG$ position where $\frac{d\delta_r}{dn} = 0$.

Proposed method for Estimating $N_0$ and $N_m$

Consider the short period perturbation dynamics of an aircraft about a trim condition represented as a time invariant linear system in state variable form:

$$\dot{x} = Ax + Bu, \quad y = Cx$$

where

$$x = [\alpha q]^T; \quad u = [\delta_e]^T; \quad Y = [\alpha q N_2]^T,$$

are the state, control and output vectors respectively with prime (') indicating transpose operator and $\alpha$ - angle of attack, $q$ - pitch rate, $\delta_e$ - elevator deflection, $N_2$ - Normal Acceleration at $CG$. The respective matrices are given by

$$A = \begin{bmatrix} \frac{Z_a}{U_0} & 1 \\ \frac{M_\alpha}{M_q} \end{bmatrix}; \quad B = \begin{bmatrix} 0 \\ M_{\delta_e} \end{bmatrix}; \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

where $Z_a$, $M_\alpha$, and $M_q$ are the aircraft dimensional stability derivatives. It is to be noted that $M_q$ definition above includes the effect of $M_{\delta_e}$, $M_{\delta_e}$ is the dimensional control derivative. $U_0$ is the trim longitudinal velocity. $g$ is the gravitation constant. The short period mode of the aircraft is given by the characteristic polynomial

$$s^2 - \left(\frac{Z_a}{U_0} + M_q\right)s + \left(\frac{Z_a}{U_0} M_q - M_\alpha\right) = 0 \quad (4)$$

where $s$ is the laplace operator. This is in the form of a standard second order system with a characteristic polynomial

$$s^2 + 2\xi\omega_n s + \omega_n^2 = 0 \quad (5)$$

where $\xi$ is the short period damping factor and $\omega_n$ is the short period natural frequency. From eqns (4) and (5) the relationship of the short period damping and natural frequency in terms of the dimensional stability derivatives are readily derived.

It is now possible to establish relationship between the static stability conditions given by eqns (1) and (2) with those of eqn (4). From eqn (1) we have

$$\frac{dC_{m}}{dC_L} = \frac{C_{m_a}}{C_{l_\alpha}}$$

The stability condition of eqn (1) implies that the nondimensional derivative $C_{m_a} = 0$ and from eqn (4), this means that the dimensional derivative $M_{\alpha} = \frac{S\bar{\alpha} C_{m_a}}{1_y}$ is zero, where $\bar{\alpha}$ is the dynamic pressure, $S$ is the aircraft reference
area (wing) and \( I \) is the pitch inertia. The stability condition of eqn (2), namely maneuver point, indicates the CG location at which aircraft stability is lost under maneuvering conditions, and this implies one of the roots of eqn (4) is zero at that limiting CG position or

\[
\left( \frac{Z_a}{U_0} M_q - M_a \right) = \omega_n^2 = 0
\]

The above analysis shows that if the elements of the "A" matrix of eqn (3) are determined by flight tests, using parameter estimation techniques, then the Neutral Point \( N_0 \) is given by the location of CG where \( M_a \) vanishes and the Maneuver point \( N_m \) is given by the location of CG where \( \omega_n^2 \) is zero.

From a practical flight test perspective this result has significant merits namely: i) Computation of \( N_0 \) is not dependent upon mass (as in the classical method) or inertia of the aircraft. ii) For determination of \( N_0 \) the dynamic maneuvers required to perform parameter estimation analysis are far simpler and require less flight test time compared to the classical method, and iii) Since \( N_0 \) and \( N_m \) estimates are derived from a knowledge of the short period natural frequency and damping, accurate determination of \( \omega_n \) and \( \xi \) from flight trajectory is possible, especially using only inertial sensors \((q \text{ and } N)\) where as in classical methods the results are dependent on air data sensors \((C_L)\) which are difficult to calibrate accurately and weight data which can only be estimated at the reference flight test point.

It is shown, in the next section, how aircraft parameter estimation techniques can be used to compute the elements of the \( A \), \( B \) and \( C \) matrices of eqn (3) and consequently \( N_0 \) and \( N_m \). The proposed method, of course, is valid provided the estimation algorithm yields unique values for the elements of the "A" matrix in eqn (3). By noting the number of free parameters in matrices \( A \), \( B \) and \( C \) and correlating with the number of poles and zeros and gains to be simultaneously estimated in the \( \delta_e \), \( \delta_e \) and \( h_T \), the parameters of eqn (3) can be estimated.

### Aircraft Parameter Estimation Method

Figure 1, gives the basis of the aircraft parameter estimation method. The aircraft is perturbed from its trim condition by applying pilot inputs to the control surface. Flight trajectories of specified aircraft response variables \( y \) (for example \( a_\alpha, q, \text{ and } N_z \)) along with the control inputs \((\delta_a)\) are recorded. A mathematical model of the aircraft dynamics is postulated in the form of eqn (3) and the mathematical model responses \( y_m \) \((a_\alpha, \text{ and } N_z)\) are generated using the same pilot input. The error between the model response and the actual aircraft response \( (y, \text{ and } -y) \) is iteratively reduced by progressively modifying the model parameters \((A, B \text{ and } C)\) matrices of eqn (3) till the error \( e = (y_m - y) \) is reduced below a specified threshold. The converged parameters of the "A" matrix yield the desired parameters \( M_a \text{ and } \omega_n \) required to estimate \( N_0 \) and \( N_m \). Many algorithms exist to perform the above estimation procedure. In this report an algorithm developed in Ref. 2, which uses the maximum likelihood estimation (MLE) criterion is used. The algorithm enjoys excellent statistical properties and also estimates the standard deviations in the estimated parameters which establish the confidence level of the parameter estimates.

### Flight Test Method using MLE to Estimate \( N_0 \) and \( N_m \)

The aircraft is trimmed for straight and level flight at different \( C_L \) (different speeds). A doublet pulse (a bidirectional pulse) input is given to the elevator. The doublet input ensures that the phugoid mode is suppressed and only the short period mode is excited. This permits the use of a short period mode approximation of the aircraft dynamics as given in eqn (3). The experiment is repeated for different CG locations. Using the MLE algorithm, \( M_a \) and \( w_a \), are computed for each CG location. Using graphical procedures, as in the conventional method, the
CG locations at which $M_\alpha$ vanishes (Neutral point $N_0$) and $\omega_n^2$ vanishes (Maneuver point $N_m$) are determined.

**Simulation validation of the new Flight Test Method**

In this section the validity of the proposed method is established using a six-degree-of-freedom (DOF) non-linear simulation of a generic high performance fighter aircraft. A special purpose software called "A Linearising Link Software" (ALLS) is used to generate the conventional flight test procedure data for computing $N_0$ and $N_m$. Using the six DOF non-linear aircraft simulation, the MLE flight test method is simulated to derive estimates of $N_0$ and $N_m$ by the proposed method and the results are compared.

**The "ALLS" Software**

The "ALLS" software was originally developed to derive linear perturbation aircraft models from a six DOF non-linear simulation. The basic principle used in the software is to define appropriate "TRIM" conditions mathematically and iteratively manipulate the control settings of the six DOF aircraft model (throttle, elevator, aileron, rudder etc.) until the defined "TRIM" aircraft state is achieved. For example, if the aircraft is to be trimmed for straight and level flight at a reference altitude and speed, the "TRIM" criterion is that all the translation and rotational accelerations must be zero and using an optimisation algorithm the "ALLS" procedure computes the control settings to achieve this condition. In the case of the pull-up maneuver, the control settings are computed such that the specified load factor (n) is achieved at the reference speed and altitude. This trim state results in a non-zero steady state pitch rate and constant angle of attack and speed conditions, which is exactly the trim condition the pilot attempts to achieve in pull-up flight tests (as desired earlier). Thus using the "ALLS" software all the data that will be required to compute $N_0$ and $N_m$ using the conventional flight testing method can be derived. Further the "ALLS" software also generates linear perturbation models, about the trim state, in the form of eqn (3), which can be used as "TRUTH" models to validate the MLE derived models.

**Simulation Results**

Using the "ALLS" software the conventional method flight testing data are generated. Fig. 2 shows the trim curves for straight and level flight (1-g) conditions (plot of $C_L$ vs $\delta_e$). The trim curves are generated for four CG locations covering a range of $C_L$ values. Notice that the trim curves are not linear with respect to $C_L$ and thus the slope $\frac{d\delta_e}{dC_L}$ is a function of the reference $C_L$ at which the neutral point is to be determined. Accordingly the local slopes are computed for two reference $C_L$ values, namely 0.093 and 0.18. Fig. 3 shows the trim curves for the pull-up maneuver as a function of load factor. The trim curves are generated for four CG condition. The slope $\frac{d\delta_e}{dn}$ for the two reference $C_L$ conditions can be computed from this figure.

The aircraft is initially trimmed at a reference $C_L = 0.18$. A doublet input is given to the elevator and parameter estimation experiments are conducted. Using the six DOF non-linear simulation of the aircraft, the angle of attack, pitch rate and normal acceleration trajectories for this input is generated. Using this trajectory data, the MLE estimation procedure is invoked by postulating a mathematical model as in eqn (3) to estimate the elements of the $A$, $B$ and $C$ matrices. Further using the "ALLS" software, linear perturbation model for the reference $C_L$ (entries of $A$, $B$ and $C$ matrices) are also generated. Assuming that the "ALLS" model is the "TRUTH" model, Table 1 establishes the achievable accuracy of the parameter estimation procedure.

It is seen from the table that the match between ALLS and MLE values for the parameters of interest namely, $M_\alpha$ and $\omega_n$ are quite satisfactory. This simulation experiment validates that the parameter estimation technique yields accurate values of the critical parameters required in the estimation of $N_0$ and $N_m$.

Fig. 4 compares the MLE method and the classical method for estimating $N_0$. Excellent agreement is seen between both the methods at the
two reference $C_L$ conditions. Fig. 5 shows the comparison of MLE and classical method to predict the Maneuver point. The agreement for $C_L = 0.093$ is very good. However there is a small discrepancy for $C_L = 0.18$ (1.5 percent $F$). A closer look at this difference reveals that in the classical method, appreciable $C_L$ excursions are required to generate the required load factors (1 to $2g$; Fig. 3) in the pull-up maneuver. Thus the non-linear $C_L$ vs $\delta_e$ (as in Fig. 2) comes into picture and the measured slope is no longer a local slope. This results in a slight error in estimation of $N_m$. However the MLE method does not have this limitation because the MLE maneuver used to generate the trajectory data is essentially a small perturbation around the reference $C_L$.

Conclusions

A new flight test and analysis method, based on system theoretic concepts, to estimate aircraft longitudinal static and dynamic stability in terms of neutral and maneuver points, is proposed. It is shown that modern parameter estimation techniques can be effectively used to compute these stability parameters. Since the stability information is extracted from the short period dynamic response of the aircraft, substantial flight test time reduction results when compared to the conventional steady state flight test procedures. Since the proposed method does not use air data information or Mass/Inertia data, the resulting estimates of the neutral and maneuver points are generally more accurate.

References


### Table 1. Comparison of ALLS and MLE Methods for Computing Stability Derivatives

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# Percent Standard deviation
1. Maximum Likelihood Estimation Procedure

Fig 1. Maximum Likelihood Estimation Procedure

Fig 2. Level Flight Trim Curves

Fig 3. Pull-up Maneuver Trim - $C_L = 0.18$

Fig 4. Pull-up Maneuver Trim - $C_L = 0.093$
Fig 4. Comparison of Classical and MLE Methods
  Neutral Paint

Fig 5. Comparison of Classical and MLE Methods
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The Maneuver Point, $N_m$ is defined as the CG position at which, under steady pull-up maneuvers, (in which the velocity and angle of attack are held constant)

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where "n" is the load factor, defined as the ratio of Lift to Weight. $N_m$ is again defined as a percentage of $\bar{c}$. The distance$(N_m - \bar{x}_{CG})$ is called the manoeuvre margin. It should be pointed out that under accelerated flight condition additional stability accrues due to pitch rate damping and thus $N_m$ is invariably aft of $N_0$.

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* Scientists, Mechanics & Div.,

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