linearizing the equations of motion about an equilibrium point for up and away flight can be extended to the ground roll phase.

The algorithms have been implemented on a ground simulator. In design cycles, the flight simulator is brought in at an advanced stage where the usefulness of its results in optimising the design is limited. By introducing the flight simulator in the design stage and implementing the trim algorithms presented in this paper, the aircraft designer is fed with more reliable information, which can be used to design.

These trim algorithms are also very useful in initialising the aircraft to a istic and user specified flight condition on ground for a take-off or landing simulation. This is helpful for validating the landing gear modelling across various platforms against a given template response. Actual flight data could also be used as the template. Previously stored time histories the same simulator are also sufficient to regenerate the response.

The conditions for quasi-steady equilibrium of aircraft on the ground are derived in Section 1 of this paper. The equilibrium point of this set of equations can be obtained by standard numerical constrained optimisation techniques. The current practice in aircraft industry for rake-off analysis makes several simplifying assumptions such as linear and consideration of only longitudinal degrees of freedom, resulting in simple formulae used in the design stage. An example of this is presented in Section 2 of this paper. As preliminary design progresses and more aerodynamic data becomes available, the designer may wish to refine his calculations. This may not be an easy undertaking, the reason being the high complexity of the aerodynamic database of a typical If the algorithm is not general enough, asymmetric operating conditions on rake-off (e.g., one engine fail, cross winds etc.) cannot be tackled. Therefore, there is a need for a general trim algorithm for obtaining the flight mechanics parameters of interest, which can work with any database without assumptions.

All the simulations and trim algorithms presented in this paper have been coded on the ELS (Engineer-in-the-Loop Simulator). ELS at the Flight Mechanics and Controls Division, NAL, is a six degree of freedom, pilot in the loop real time simulation facility. the ELS has been configured to simulate a single engine, tailless delta wing fighter aircraft (henceforth called the GFA).

The rigid body dynamics of aircraft is governed by six degrees of freedom, namely the three translations and three rotations along the spatial co-ordinate$. resulting equations[3] can be referred in standard textbooks. general aircraft dynamic equations which non-linear in nature can be cast in the following implicit form'.

where $\bar{f}$ is a vector n non-linear functions f_i. $\bar{X}$ is state Vector β ϕ ψ P_s Q_s R_s X_E $H_E]$ and is the input vector U^T δ_t δ_e

The equilibrium or singular point(s) of (1) satisfy the following condition.

$$\dot{\bar{X}} = 0 \text{ for some given value } . \qquad (2)$$

under the assumptions of constant aircraft mass, flat earth approximation and neglecting atmospheric density effects on aircraft motion, the equations of motion allow us to decouple the earth co-ordinates closed set represented by (1). Thus, we obtain set of nine first order differential equations. The vector is now given by $X^T = [V_T \quad \alpha \quad P_s \quad Q_s \quad R_s \quad \theta \quad \psi]$. purposes of aircraft performance, stability analysis and control law design, the aircraft motions need to be analysed for various manoeuvres (straight level flight, turning flight, pull-up , push over etc.). To obtain the commonly used equilibrium points for flight mechanics to further reduce the state vector from nine to six, to aircraft degrees of freedom ($X^T = [V_T \quad \alpha \quad P_s \quad Q_s \quad R_s]$ with constraints on either the euler angles ( ) and/or their rates. Therefore (2) implies the following conditions to be satisfied for equilibrium.

$$\dot{V}_T, \dot{\alpha}, \dot{\beta}, \dot{P}_s, \dot{Q}_s, \dot{R}_s \quad 0, \bar{U} \equiv \text{constant} \qquad (3)$$

subject to appropriate the constraints. The constraints applied depend upon the type of flight mode required[1] (straight and level, level turn, pull up etc.).

Based on (3), *one can* derive equilibrium **conditions** for **the various phases *of* the** rake-off manoeuvre (Fig. la). These will be discussed in **the** following sections.

For deriving **the trim** point **conditions** manoeuvre **has been** divided into three **phases as shown** in Figure lb. They are · **accelerated ground** roll, the nose wheel **lift-off** point and **rotation** to lift-off pitch attitude. In what follows, these **phases are** examined separately and appropriate **trim** conditions defined.

a. Accelerated <u>ground</u> with thrust *set* <u>to</u> fixed value (usually **at max.** dry

accelerating **on the** runway, all **the** six accelerations **terms** in (3) are non **zero. A** reasonable estimate ***of* the** equilibrium **state *of* the** aircraft is obtained if **the** V_r **equation** is ignored. For this phase ***of*** flight all **body** axis angular rates are close to **zero.** Further, **the** angle (ϕ) and **the** yaw angle (ψ) are **also** equal to zero. Thus, ***we*** have **the** following **conditions** to **be** satisfied at trim point.

The *case* ***of*** asymmetries arising out of single engine failure where above assumptions are violated, is examined in the concluding **part** of **Section** 1.

Equations (4a,b) are in **the** form ***of*** a ***constrained*** minimisation with equality constraints. The algorithm is suitable for **this** problem. The following is a commonly **used** cost **function.**

In order **to** satisfy each of **the** equations in (4a), ***we*** need **to** select **the** five control variables. The choice ***of*** these variables should be such that each **equation** in (4a) is strongly influenced **by** at least ***one*** different variable. At **the** same **time** these control variables must ensure a unique solution. **To** select such variables ***we*** appeal **to the** physics ***of* the** problem. In up and away flight, usual control variables[1] **are** $\dot{\alpha}$ equation is

$\dot{Q}$, thatof δ_e, $\dot{P}$, thatof **6,** and the $\dot{R}$, equationisa strong function of 8, **.** In **the** *case* ***of*** aircraft resting on **both** wheels, ***a*** depends on θ (constraint (4b)), which in turn depends on **the** force and moment balance at equilibrium ***(see*** Fig. **2).** Also in ***cases*** δ_e, **the** elevator angle is preset during ground roll. Therefore, ***we* need to** replace ***a*** and δ_e with some other **appropriate** control variables. The obvious **choice** is **the** forces **on the nose** (Fnose) and main (Fmain) wheels respectively making **as the** set ***of*** control variables. Thus ***we*** have to ***satisfy*** (4a) **subject to** constraints (4b), **for** fixed values of the mach number, elevator setting **6, .** *and* throttle setting **6**

When the aircraft reaches **rotation *speed*** V_R **,** **the** pilot applies back **stick** (up elevator) to ease aircraft nose up **to the desii** rake-off attitude. Between the time he **starts** rotation and **the** point **at** which nose wheel lifts ***off* the ground,** a **trim** condition ***can*** be defined which **satisfies** (4). Therefore, ***we can*** use **the trim** algorithm described above to ***obtain* the locus *of* such trim points as** a **function *of* the stick** deflection (***or*** equivalently **the** elevator deflection).

b.

: **For** determination ***of* the** exact point ***of* nose** wheel lift-off, further **conditions** need to be satisfied. **These** conditions **are as** follows.

i. reaction force **on** nose wheel is (Fnose = **0)**

ii. Nose tire is just touching the ground (in undeflected state).

The two constraints in (6) have to be imposed in addition to those specified in **(4b). Since** Fnose is now identically **zero,** elevator **deflection *can*** be taken **as** a control variable. Thus for nose wheel-off **trim, the condition** (4a) **has** to be satisfied subject **to the** constraints specified in (4b) and ***(6).* The** control

number **and** throttle setting δ the user *can* determine **the** minimum elevator required to just lift **nose** wheel off the ground. In aircraft preliminary design, the designer may want **to** determine **the** rotation **speed** for a given maximum elevator deflection. In such a *case,* ***one can*** replace elevator deflection with mach number **as** a control variable. Then for a given maximum

elevator .
minimum speed at which rotation be initiated.

c.

wheel off : For the of nose wheel ground, the an? same as in (4b) and the equations to be minimised are as in (4a). Since the nose is ground, the control chosen F_{main}

In case of arising out of engine or single control surface failures, the aileron 6, be set to a fixed value, while F_{main} can be split into two forces (one each for either of the gear struts) F_{port} and $F_{starboard}$. Also the angle (ϕ) must be reset every cycle depending on the values of the three landing gear farces F_{nose}, F_{port} and $F_{starboard}$. This concludes the derivation of the trim algorithms for the complete p o d roll In the next section equations for trim on ground derived.

The approximate equations for aim ground can be derived by considering the simplified pitch plane equations only (Fig. 3). The force and moment balance equations as follows.

The lift, and pitching moment terms can be expanded using the aerodynamic derivative formulation. There are three equations and an equal number of unknowns (θ, R and 6,) provided forward forward acceleration and the thrust are fixed. (7) above is poorly conditioned with respect to the pitch attitude θ. The remaining equations (8) and (9) can be solved for R and 6,. provided θ is known. Using (8) and (9), an estimate of the elevator required to hold a given pitch attitude while rolling on the ground at a given and thrust setting can be computed. This computation is sensitive to the values of aerodynamic derivatives used, (particularly $C_{m\delta e}$) as well as the distances x_m z_m

In order to calculate the minimum elevator required to just lit the nose wheel the ground, one more relation between the aircraft attitude and R or δ, This can be obtained the geometry in Figure 4.

. vs. force characteristics is over the range of l_m is related to l_{mo} and normal reaction force R in a direct way as follows.

factor of two in equation (11) for the of two main gears on the minimum elevator required to lift the nose wheel off the ground, main wheel force and pitch attitude at nose wheel lift-off point be found using equations (8), (9) (10) and (11) after a few iterations. The results of this method are with the exact values in Section 4. In the next section issues for the exact formulation of Section 1 discussed.

aim constraints explained in 1 can be implemented with minor modifications to the up and away trim For aircraft on ground, the farce and moment contributions arise from the following - aerodynamic forces and moments, propulsive forces and moments, gravitational forces and ground reaction forces and moments (due to contributions from propulsion, and gravitational forces and moments are already computed io the up and away algorithms. The flowchart for calculating the forces and moments due to the landing gear model in aircraft body axis is presented in Figure 5. This portion of the code is common to all the algorithms described above. it be coded as a separate module.

The values obtained by the approximate method outlined in Section 2 are compared with the 'exact' values computed by the algorithm described in Section 1.1c in Table 1 for the GFA. It is seen that for all the values in Table 1, error is within 1deg.

The results of the computations of the minimum elevator point using equations of Section 2 are compared with those obtained from trim algorithm described in Section 1.1b in Table 2 for various configurations of the GFA. Again the agreement is good for the predicted elevator deflection (within 1.5deg) and the pitch attitude at nose wheel lift-off (within 1deg). The normal reaction force is predicted with a fair degree of accuracy (within 6% of total weight). This is due to the involved in representing the main landing gear by an equivalent spring (Fig. 4).

The results of the trim algorithm for the GFA are plotted in Figure 6 in the pitch attitude vs. elevator deflection plane for various speeds of rotation 220kmph and 240kmph). The initial decreasing segment of each curve corresponds to accelerated ground roll on all wheels. The lowest point of each curve corresponds to the nose wheel lift off point. The increasing segment of each curve to the rotation upto θ_{LOF} (which for this aircraft is 13deg) after nose wheel lift-off. The maximum up elevator deflection required reflects the static stability (for a given speed) that must be overcome in order to lift the wheel off the ground.

In Figure 8 a typical simulation result of a take-off run is presented. The GFA is actually unstable in pitch in the low mach number range. It has been stabilised by the use of pitch rate and normal acceleration feedback. It is clear that the nose wheel (EV04) leaves the ground just as the elevator crosses the minimum elevator required point. The rotation was initiated by the pilot at about 210kmph with a back stick (PSTICK) of roughly 75% of full back stick deflection (42mm). The plots in the second column of Figure 8 show that take-off occurred at about 10sec (EV03).

In Figure 7. the results of the simulation (as presented in Figure 8) are plotted with the results of trim analysis (Fig. 6) in the θ vs. elevator deflection δ_e plane. The simulation results are quite close to the trim calculations, since, the pilot is attempting to execute a smooth rotation manoeuvre and therefore, he needs to apply only an incremental elevator deflection over and above that which is already required to overcome the static stability (Fig. 6). For a non-zero pitch rate, more elevator deflection over and above that shown in the trim plots of Figure 6 is required. Exactly how much more will obviously depend upon the control effectiveness of the aircraft in pitch and its inertia.

An extension of the equilibrium trim analysis to the ground roll phase has been proposed. Its applicability has been demonstrated by comparison with six degree of freedom simulation of a typical fighter aircraft. It is argued that due to the nature of the take-off rotation manoeuvre, the proposed trim strategy is equally applicable to all class of aircraft and for all types of control mechanisms (conventional or fly-by-wire). The comparison also shows that linearisation of the equations of motion in the take-off phase is meaningful. An approximate method for the computation of the parameters of interest to the aircraft design engineer is also presented. This method can be used in the preliminary design stage and gives reasonably accurate results. The advantages of the approximate method is that it relies on a minimum of information about the aircraft and landing gear parameters. It is noted that a simulator platform is the ideal environment for the analysis and application of the trim algorithms presented in this paper.

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1. Exact and Elevator as Function of Pitch

Table 2. Exact and Predicted Parameters at Nose Wheel Lift-off Point

* Indicates the values estimated from approximate equations of *section* 2.

Accelerated Ground
'GROUNDRUN' Trim

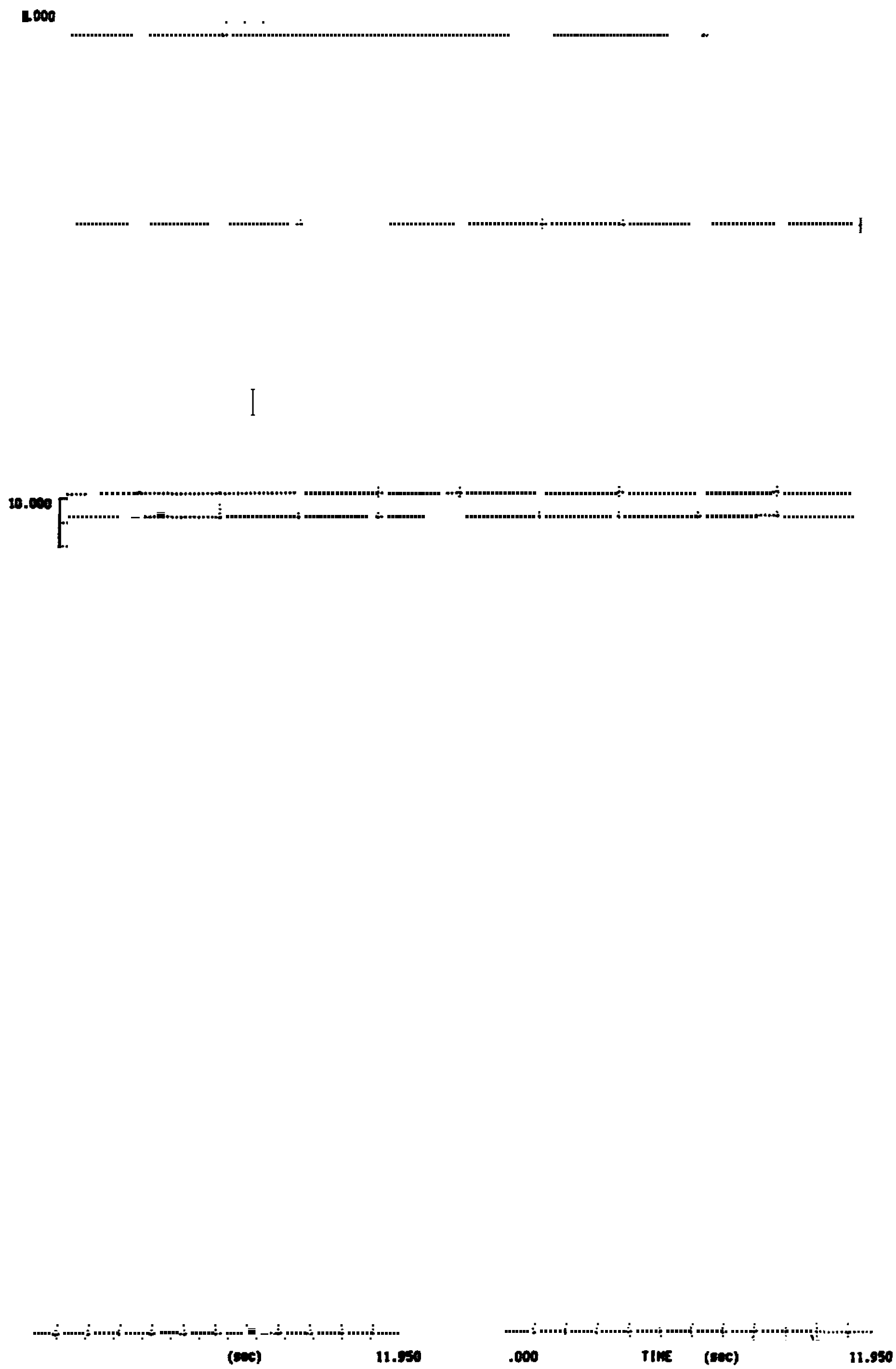
Figure lb. of a Take-off Manoeuvre

2. Forces and Moments on the Aircraft on Ground

Figure 4. **Spring Representation** *of* **Main Landing** Gear

P

5. **Flow** **of Trim** Constraints **Routine**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