Active flutter suppression of a cantilever plate at supersonic speeds

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Modern high-speed aircrafts are flown using the fly-by-wire system. The dynamics of automatic flight control system interacts with aircraft's structural dynamics and aerodynamics to give rise to aeroservoelastic problems. Such problems can be avoided using an active control system. As a feasibility study, active flutter suppression of a cantilever plate wing was carried out. In this study it was demonstrated that the flutter Mach number of the cantilever wing can be increased by 12.5%.

Notations

- $\alpha$: Constants defined in Equation (20)
- $A$, $A_0$: Co-efficient matrix in state-space form
- $A_R$, $A_I$: Real and imaginary parts of unsteady aerodynamic force coefficient matrix
- $A_{R_e}$, $A_{I_e}$: Real and imaginary parts of unsteady aerodynamic force coefficient matrix associated with control surface motion
- $b$: Reference semi-chord
- $C_k$: Constants defined in Equation (19)
- $D_k$: Constants defined in Equation (22)
- $I$: Unit matrix
- $k$: Reduced frequency
- $K$: Stiffness matrix
- $K_a$: Control system gain
- $K_{a_e}$: Coupling stiffness matrix for control surface degrees of freedom
- $M$: Mach number
- $[M]$: Mass matrix
- $M_{se}$: Coupling mass matrix for control surface degrees of freedom
- $p(s)$: Polynomial in s, defined in Equation (11)
- $\Omega_e$, $\Omega$: Real and imaginary parts of unsteady aerodynamic force coefficients matrices
Introduction and literature survey

During flight through atmosphere, the structural components of aerospace vehicles deform in a time-dependent fashion and produce motion-dependent unsteady aerodynamic forces which are responsible for dynamic aeroelastic instability—flutter, which leads to catastrophic failure of vehicle components. This occurs due to interaction between elastic, inertia and unsteady aerodynamic forces. Several passive methods were being used in the past, but the recent technological advances in the field of control theory and availability of highly reliable control system electronic hardware offer a new way of treating the problem of flutter instability. This approach consists of a rapidly responding control system which is actuated by the motion of the main surface and which leads to appropriate deflection of the control surface. In this way, when the control surface aerodynamic forces are used to suppress flutter instability; this is known as active flutter suppression and the whole system is referred to as active control system (ACS). Studies have shown that sizeable weight savings are possible by using active flutter suppression as opposed to the normal passive flutter control method. In application, active controls employ various sensors, appropriately located on the vehicle to detect the deformation or motions, with suitable servo feedback to accomplish the desired objectives. Figure 1a, shows a simplified general schematic; there can be multiple inputs and outputs, separate or combined systems, and onboard microcircuit computers.

Active control technology implies the intentional use of an aircraft of automatic control system. The system involves feedback control rather than passive aerodynamic design feature and is used to drive a number of specific control surfaces and possibly auxiliary force and moment generators. These controls improve the structural behaviour...
Active flutter suppression of a cantilever plate at supersonic speeds

![Block diagram and interaction triangle]

Figure 1. (a) Block diagram of the active control system; (b) aeroservoelastic interaction triangle.

Elements of aerospace vehicles deform under the influence of unsteady aerodynamic forces produced by changes in stability – flutter, which leads to interaction between elastic, aerodynamic and control system characteristics and which leads to appropriate control system design. Figure 1a shows a simplified block diagram and Figure 1b shows a topological representation of the interaction between aerodynamic, elastic and control system elements. The block diagram in Figure 1a is also used for analytical studies and for the purposes of system description. The figure shows elements of aerodynamics, the aerodynamic system and control system, which are connected to each other by input (u) and output (v) variables. The control system is represented as a black box with the following input variables: actuator (a), control law (c), and actuator (d).

Aerodynamic systems (A), structural dynamics (S) and control system (K) are strongly related. They can be viewed as a closed-loop system with feedback elements, and they interact through their transfer functions. The block diagram in Figure 1a shows the interaction between the aerodynamic system and control system, as well as the interaction between the control system and structural dynamics. The figure also shows the interaction between the aerodynamic system and structural dynamics, as well as the interaction between the control system and structural dynamics.

Figure 1b shows a topological representation of the interaction between aerodynamic, elastic, and control system elements. The figure shows that the interaction between these elements is complex and cannot be easily described using simple block diagrams. The figure also shows that the interaction between these elements is not linear and that it is not possible to use simple transfer functions to describe the interaction between these elements.

The active control system refers to a system that directly produces a change in the motion of the aircraft. This includes automatic (no pilot's action required) as well as manual systems. The systems or devices referred to may be flight critical, i.e., necessary for continued safe flight. Analytical ACT studies have indicated that the most significant performance improvements are achieved from six control functions:

1. Augmented stability (AS)
2. Gust load alleviation (GLA)
3. Maneouvre load control (MLC)
4. Fatigue reduction (FR)
5. Ride control (RC)
6. Flutter mode control (FMC)

A good description of these is given by Shomber et al. The flexibility of aircraft not only leads to several aeroelastic problems, but also large amplitude motion due to structural deformation which is comparable with rigid body motion like pitch and plunge. Most of the recent configurations are statically unstable and
are controlled by automatic flight control systems. For these aircraft, the frequencies of
the lower elastic modes and rigid-body modes are quite close. They interact in the
presence of actuator mechanism of the flight control system (or active control system) and
lead to instability known as aeroservoelastic (or ASE) instability. The ASE instability
occurs due to interaction between an aircraft structural dynamics, aerodynamics and
automatic flight control system (Figure 1.5). The ASE entered into the aircraft design with
the increased demand for minimum structural weight to meet strength requirements and by
the use of high-authority, high-response, automatic flight control system to improve
aircraft performance, stability, service life, ride quality, etc.

Radovich has classified the aeroservoelastic problems in the following way:

1. Steady aerodynamic, low-order servodynamics, structural elasticity in stability
derivatives.
2. Steady manoeuvre involves steady aerodynamics, zero-order servodynamics and
structural elasticity.
3. Unsteady aerodynamics, high-order servodynamics and accurate modelling of
structural elasticity and aircraft inertia.

The aeroservoelastic models are the analytical glue for all feedback control system
and constitute the ACS’s database for simulating the aircraft dynamics. The data cover
structural deflection, aerodynamics, control surface details, sensor characteristics,
servodynamics and active control computer definition, including control fluctuations and
monitors. This paper deals with one of the ASE problem, namely, active flutter
suppression.

For the design of an active control system, various approaches can be used. In the
case of active flutter suppression system, the classical design technologies are based
either on the application of the root locus method, frequency response method and
optimal control theory techniques. There are other less conventional design techniques
such as Nissim’s aerodynamic energy concept and the method of fictitious structural
modifications which can be applied as well.

For the design of an active flutter suppression system, it is necessary to formulate the
equations of motion in Laplace domain. The structural-dynamic equations can be written
in Laplace domain easily. Unsteady aerodynamic forces are normally calculated in
frequency domain for a particular combination of Mach number ($M$) and reduced
frequency ($k$). Therefore, the main problem in deriving equations of motion for active
flutter suppression is to convert the tabular form of aerodynamic data into continuous
function in the Laplace domain. Several methods are available in the literature for this
purpose. The cumbersome process of getting unsteady aerodynamic loads could be
avoided in supersonic regime if piston theory is used. While using this theory, the
complex generalized force coefficient matrix is evaluated for a given value of $M$ and $k$.
For fixed value of $M$, the real part remains constant but the imaginary part varies linearly
with $k$. When the imaginary part is multiplied by $1/k$, it remains constant for all $k$. In this
way, at constant altitude, the unsteady aerodynamic force coefficients are made to vary
with Mach numbers. In this paper a cantilever plate with control surface is considered for

**Theoretical analysis**

**Equations of motion in Laplace domain**

The equations of motion can be written:

\[
[M] \{q\} + [K] \{q\} \rightarrow 0
\]

or

\[
[M] \{\dot{q}\} + [K] \{q\} = 0
\]

Let

and

Now Equation (2) can be written as
active flutter suppression study at supersonic speeds. The control system characteristics and actuator dynamics are represented similar to that of Reference 17. The equations of motion are derived in state-space form with and without active control system. Flutter characteristic of this configuration has been studied in References 18 and 19 without application of a control system. The flutter stability margin is determined using the root locus technique. The present study is an extension of the above studies for active flutter suppression.

**Theoretical analysis**

**Equations of motion without the control system on**

The equations of motion of lifting surfaces in flight in modal coordinate are given by

\[
[M] \{q\} + [K] \{q\} = [Q] \{q\} \tag{1}
\]

where \([M]\) is the generalized mass matrix (diagonal), \([K]\) is the generalized stiffness matrix (diagonal) and \([Q]\) is matrix of generalized aerodynamic force coefficients. Now, Equation (1) can also be written in the following form:

\[
[M] \{q\} + [K] \{q\} = - (1/2) \rho U^2 [\Theta_0 + \Theta_1] \{q\}
\]

\[
= - (1/2) \rho U^2 [\Theta_0] \{q\} - (1/2) \rho U \frac{U}{\omega_0} \Theta_1 \Theta_0 \{q\} \{i \omega q\}
\]

or

\[
[M] \{q\} + [K] \{q\} = - [A_0] \{q\} - (1/2) \{A_1 \{q\}\}
\]

or

\[
[M] \{q\} + [K] \{q\} + [A_0] \{q\} + (1/2) \{A_1 \{q\}\} = 0 \tag{2}
\]

Let

\[
\{q\} = (1/2) [M^{-1} A_1] \{\dot{q}\} + [M^{-1} K + M^{-1} A_0] \{q\}
\]

and

\[
\{\dot{q}\} = x \tag{3a}
\]

and

\[
\{q\} = x \tag{3b}
\]

Now Equation (2) can be written in state-variable form as

\[
\{\dot{x}\} = -(1/2)[M^{-1} A_1] \{x\} - [M^{-1} K + M^{-1} A_0] \{q\} \tag{4}
\]
Combining Equations (3a) and (4) one can write

\[
\begin{bmatrix}
0 \\
I
\end{bmatrix} = \left[ -M^{-1}K - M^{-1}A_R - (1/k)M^{-1}A_L \right] \begin{bmatrix}
q \\
x
\end{bmatrix}
\] (5)

The above equation can be written as

\[ X = AX \] (6)

where

\[ A = \begin{bmatrix}
0 & I \\
-M^{-1}K - M^{-1}A_R - (1/k)M^{-1}A_L & 0
\end{bmatrix} \] (7)

Equation (6) is solved as an eigenvalue problem for a range of Mach numbers. Since \( A \) is unsymmetric, the eigenvalue of \( A \) is in general complex. They are calculated using some standard subroutine on unsymmetric matrix eigenvalue extraction.

**Equations of motion with control system on**

In this section, the equations of motion is derived for the lifting surface with an active control system. To achieve this, it is necessary to incorporate the actuator transfer function and an appropriate control law into the equation of motion. The block diagram of the active control system is shown in Figure 1a. The frequency response function of the actuator in Laplace domain is given by

\[ \frac{q_a}{q_c} = \frac{K_p}{(s^2 + 2\alpha_0^2s + \omega_0^2)(s^2 + 2\alpha_1^2s + \omega_1^2)} \] (8)

where \( q_a \) is the control surface deflection, \( q_c \) is the control command to the actuator and \( K_p \) is the control law gain parameter. The next step is to use a control law to demonstrate at least 10% increase in critical flutter Mach number over that of the passive wing. For this purpose, the transfer function of the control system (also known as control law) is chosen as \( z = \frac{1}{s^2 + 2\alpha_0^2s + \omega_0^2} \) (9)

Therefore, the transfer function of the active control system as a total is given by

\[ \frac{q_a}{q_c} = \frac{q_a}{q_c} \times \frac{1}{z} \] (10)
Active flutter suppression of a cantilever plate at supersonic speeds

\[ \frac{K_0}{(s^2 + 2\alpha_0 U_s + \alpha_0^2)(s^2 + 2\alpha_0 U_s + \alpha_0^2)(s^2 + 2\alpha_0 U_s + \alpha_0^2)} = \frac{K_0}{p(s)} \]

(11)

Now from Equation (1) one can write

\[ [M; M_\alpha] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} + [K; K_\alpha] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} = -\left( \frac{1}{2} \right) \rho U^2 [Q; Q_\alpha] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} \]

(12)

where \( M_\alpha \) and \( K_\alpha \) are control surface inertia, stiffness and aerodynamic coupling matrices, respectively. In the present study \( M_\alpha \) and \( K_\alpha \) matrices are omitted. Equation (12) can be written like Equation (2) as

\[ [M] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} + [K] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} = -\left( \begin{bmatrix} \ddot{\theta}_R \end{bmatrix} \right) \left( 1 \right) \begin{bmatrix} \ddot{\theta}_R \end{bmatrix} \begin{bmatrix} \ddot{\theta}_R \end{bmatrix} - (1/k) [\ddot{\theta} \ddot{\theta}] \begin{bmatrix} q \ddot{\theta} \end{bmatrix} \]

or

\[ [M] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} + [K - A_{\alpha}] \begin{bmatrix} q \\ \dot{q}_\alpha \end{bmatrix} = -\left( A_{\alpha} \right) \begin{bmatrix} q \ddot{\theta} \end{bmatrix} - (1/k) [\ddot{\theta} \ddot{\theta}] \begin{bmatrix} q \ddot{\theta} \end{bmatrix} \]

(14)

For a single control surface and a single sensor (accelerometer) the control law transfer function can be assumed to be of the form

\[ \frac{Z(q_\alpha)}{Z(s)} = \frac{K_0}{p(s)} \]

(15)

or

\[ (q_\alpha) = \frac{s^2 K_0}{p(s)} z, \quad z = [T]q \]

(16)

where \( z \) can be related to \( q \) through a transfer matrix. Substituting Equation (16) into Equation (14) and the result in Equation (15), one can write

\[ \left( [M]s^2 + [A_{\alpha}]s + [K - A_{\alpha}] \right) \begin{bmatrix} q \ddot{\theta} \end{bmatrix} = -\left( A_{\alpha} \right) \begin{bmatrix} s^2 K_0 \end{bmatrix} \begin{bmatrix} q \ddot{\theta} \end{bmatrix} \]

or

\[ \left( [M]s^2 + [A_{\alpha}]s + [K - A_{\alpha}] \right) p(s) - \left( A_{\alpha} s^2 K_0 \right) q = 0 \]

(17)

Substituting for \( p(s) \) from Equation (11) in Equation (17), one can write
\[(M^6 + (C_1M - A_1)^3 + [C_2M + (K - A_2) - C_1A_1]s^4 + [C_3M - C_2A_1 + C_1(K - A_2)]s^5 + [C_4 - C_3A_1 + C_3K_0]s^6 + [C_5M - C_4A_1 + C_5K_0]s^7 + [C_6M - C_5A_1 + C_5(K - A_2)]s^8 + [M - C_6A_1 + C_6(K - A_2)]s^9\] \{q\} = 0 \quad (18)

where
\[
C_1 = a_1 + a_2; \quad C_2 = a_6 + a_5a_6 + a_1a_6; \quad C_3 = a_7 + a_5a_7 + a_1a_7;
C_4 = a_4 + a_5a_4 + a_1a_4 + a_7a_4; \quad C_5 = a_5a_7 + a_5a_1 + a_7a_1; \quad C_6 = a_6a_7
\]

and
\[
a_1 = 2\omega_6\xi_8; \quad a_2 = \omega_8^2; \quad a_3 = 2\omega_6\xi_1; \quad a_4 = \omega_2^3
a_5 = a_1 + a_5; \quad a_6 = a_1 + a_4 + a_1a_5; \quad a_7 = a_1a_4 + a_1a_5
a_8 = a_2a_4; \quad a_9 = 2\omega_6\xi_8; \quad a_{10} = \omega_2^3
\]

Equation (18) can also be written as
\[
(D_1 s^6 + D_2 s^7 + D_3 s^8 + D_4 s^9 + D_5 s^{10} + D_6 s^{11} + D_7 s^{12} + D_8 s^{13} + D_9 s^{14}) \{q\} = 0 \quad (21)
\]

where
\[
D_1 = M
D_2 = C_1M - A_1
D_3 = C_2M + (K - A_2) - C_1A_1
D_4 = C_3M + C_3(K - A_2) - C_2A_1
D_5 = C_4M + C_4(K - A_2) - C_3A_1
D_6 = C_4M + C_4(K - A_2) - C_3A_1 - A_1a_K
D_7 = C_5M + C_5(K - A_2) - C_4A_1 - A_1a_K
D_8 = C_6M + C_6(K - A_2) - C_5A_1 - A_1a_K
D_9 = C_6M + C_6(K - A_2) - C_5A_1 - A_1a_K
\]

Equation (21) can be reduced to a series of first-order equations of the following form:
\[
s (X) \{X\} = [A_s] \{X\} \quad (23)
Active flutter suppression of a cantilever plate at supersonic speeds

\[(K - A_0)\delta^5\]
\[\delta^5\]
\[\text{(18)}\]

\[(K - A_0)\delta^5\]
\[\delta^5\]
\[\text{(19)}\]

\[\delta = 0\]
\[\text{(21)}\]

Figure 2. Finite-element model of the cantilever plate surface

Figure 3. Root locus plot of flutter eigenvalue without control
Active flutter suppression of a cantilever plate at supersonic speeds

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Diagram c: 
- $K = 1000$, $\zeta = 0.7$, $\omega_n = 80$ Hz
- Indicates modes from 3.2 to 4.8
- Response at $\omega = 5$

Diagram d: 
- $K = 1000$, $\zeta = 0.7$, $\omega_n = 80$ Hz
- Indicates modes from 3.2 to 4.8
- Response at $\omega = 5$

continued
Figure 4. Root locus plot of flutter eigenvalue with control.

Figure 5. $M_c$ vs $\zeta_c$ curve.
Active flutter suppression of a cantilever plate at supersonic speeds

where

\[
\begin{align*}
X &= \begin{bmatrix}
    x^1(q) \\
    x^2(q) \\
    x^3(q) \\
    x^4(q) \\
    x^5(q) \\
    x^6(q) \\
    x^7(q) \\
    x^8(q)
\end{bmatrix}
\end{align*}
\]

and

\[
A_p = \begin{bmatrix}
D_1^{-1}D_2 & D_1^{-1}D_3 & D_1^{-1}D_4 & D_1^{-1}D_5 & D_1^{-1}D_6 & D_1^{-1}D_7 & D_1^{-1}D_8 & D_1^{-1}D_9 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

For a fixed value of air density (altitude) and velocity, the eigenvalues of Equation (23) are the roots of the characteristics equation. Root loci can be constructed which correspond to the variation of eigenvalues of the system as Mach number is varied.

Numerical calculations

For numerical calculations, a cantilever plate is considered. The geometry of the plate is shown in Figure 2. First, the flutter analysis for this configuration is made using NASTRAN package considering the control as an irreversible system (control surface is locked with main surface). In this process, the generalized mass, stiffness and aerodynamic matrices are also obtained. Three modes and the corresponding modal quantities (mass, stiffness and aerodynamic forces) are considered to formulate equations of motion and subsequent representation in state-variable form. The mass and stiffness matrices are real but the aerodynamic matrix is complex. Since piston theory is used, the real part is constant (independent of \( k \)) and imaginary part varies linearly with \( k \). In the flutter equation (Equation (2) or (14)) the imaginary part of the unsteady aerodynamic force co-efficients remains constant when divided by the reduced frequency. Thus, the final flutter equation is solved by varying the Mach number while the density of air is kept constant for a particular altitude. Then the flutter equation is expressed in state-space form and the flutter stability analysis is carried out using the root locus technique. In this,
the real part of the complex eigenvalue at each Mach number is plotted against the imaginary part. The system is stable if a point lies on the left half of the imaginary axis. A point on the imaginary axis implies that the system is marginally stable. Therefore, a point just on right side of the imaginary axis is taken as initiation of flutter.

Results and discussion

The eigenvalues of Equation (6) are the roots of the characteristic flutter equation for the cantilever plate at sea level, without flutter suppression system. The root locus for increasing values of the Mach number (air speed), from 3.1 to 4.3, is presented in Figure 3. Arrows indicate the direction of increasing Mach number. The flutter instability occurs at $M_d = 3.945$, where the, torsional mode becomes unstable.

The eigenvalues of Equation (23) are the roots of the characteristic flutter equation for the cantilever plate with a trailing edge control surface and active flutter suppression system. The inclusion of control system and actuator dynamics into the aeroelastic system changes the order of the system from second to eighth (Equation (21)).

The flutter speed with active control system on depends on three system parameters—gain of the control system ($K_a$), the frequency of actuator output ($\omega_a$) and its damping ($\zeta_a$). It was found that for some combination of $K_a$, $\omega_a$ and $\zeta_a$, the critical flutter Mach number is lower than the system without active control system. Therefore, for a parametric study, any two of the above parameters are kept constant and the third one is varied. To

![Figure 6. Root locus plot of flutter eigenvalue with control.](image)

Table 1: Variation of Flutter Mach number ($M_f$) with control system parameters.

<table>
<thead>
<tr>
<th>$K_a$</th>
<th>$\omega_a$</th>
<th>$\zeta_a$</th>
<th>$M_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1</td>
<td>50</td>
<td>0.5</td>
<td>3.85</td>
</tr>
<tr>
<td>0.2</td>
<td>60</td>
<td>0.6</td>
<td>4.0</td>
</tr>
<tr>
<td>0.3</td>
<td>70</td>
<td>0.7</td>
<td>4.15</td>
</tr>
<tr>
<td>0.4</td>
<td>80</td>
<td>0.8</td>
<td>4.25</td>
</tr>
</tbody>
</table>

In conclusion, the effect of actuator output frequency ($\omega_a$) and damping ($\zeta_a$) on the flutter Mach number is significant. Higher frequencies and damping levels result in lower flutter Mach numbers, indicating improved flutter suppression effectiveness. Further studies are needed to optimize these parameters for practical applications.
Active flutter suppression of a cantilever plate at supersonic speeds

Table 1. Variation of flutter Mach number ($M_{cr}$) with control system gain $K_c$

<table>
<thead>
<tr>
<th>$K_c$</th>
<th>$M_{cr}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10E+10</td>
<td>4.14</td>
</tr>
<tr>
<td>10E+11</td>
<td>4.496</td>
</tr>
<tr>
<td>Without control</td>
<td>3.945</td>
</tr>
</tbody>
</table>

study the effect of actuator output frequency, $K_0$ and $\omega_0$ are kept constant. It is found that critical flutter Mach number is maximum when actuator output frequency is equal to the torsional frequency of the plate. Next, the effect of actuator damping is studied. The root locus plots for different values of damping are shown in Figures 4a-e. For low values of damping the first mode becomes unstable (Figures 4a and 5) but for higher values of damping the second mode becomes unstable (Figures 4e-e). The critical Mach number for different values of damping is presented in Figure 5. It is clear from Figure 5 that the critical flutter Mach number is maximum when damping value is 0.5.

Finally, the effect of control system gain is studied and presented in Figure 6 for $\omega_c = 1.5$ and $K_0 = 10E + 11$. The critical flutter Mach number of various gain is presented in Table 1.

Conclusions

The flutter characteristics of a cantilever plate, with and without an active flutter suppression system on, are studied in this paper. Since the flutter frequency and speed (at supersonic flow) are quite high, the actuator output frequency will have to be quite high. At present such actuators are not available. Therefore, the present study is a numerical experimentation and demonstration of active flutter suppression concept. It may not have any practical utility.

References


