

## **ADAPTIVE CONTROL OF MICRO AIR VEHICLES**

---

Shaik Ismail\*, Abhay A. Pashilkar\* and Ramakalyan Ayyagari<sup>#</sup>

\*Scientist, FMC Division, National Aerospace Laboratories, Bangalore, India.

<sup>#</sup>Prof. and HoD, Department of ICE, NIT Trichy, India

### **ABSTRACT**

*This paper discusses the problems associated with aerodynamic modeling, control and simulation of Micro Air Vehicles (MAVs). It is suggested that MRAC-based adaptive control methods are more suitable than the classical PID and Robust Control methods for MAVs. The salient features of conventional MRAC methods, and two significant and recent variants of these methods – the Modified Reference Model MRAC method and the  $L_1$  Adaptive Control method, are briefly described. These adaptive control methods are applied to the roll control of a typical MAV and the results are discussed. The Sliding Mode Control, which can be considered as a robust adaptive control method, is also implemented for pitch rate tracking of the MAV for the sake of comparison.*

**Key Words:** Micro Air Vehicle, Unmanned Aerial Vehicles, Adaptive Control, MRAC, M-MRAC,  $L_1$  Adaptive Control, Sliding Mode Control

### **1. INTRODUCTION**

Micro Air Vehicles (MAVs) are being used extensively for both military missions and civil applications due to their low-cost, small size and easy portability. At present, the MAV control, guidance and navigation systems have limited processing power due to stringent requirements of size, weight, and power consumption. However, the civil and military applications require MAVs with enhanced capabilities like detecting and avoiding obstacles, tolerating unexpected flight conditions and gusts and turbulence, interfacing with different payload sensors, tracking moving targets, and operating with other manned and unmanned systems.

Further, the manufacturing processes associated with MAV airframes are not precise enough to guarantee uniformity in the aerodynamic coefficients. Also, crashes often change the aerodynamic properties of the airframe. Hence, accurate knowledge of the aerodynamic coefficients of MAVs can not be guaranteed.

Accurate and reliable sensor output is essential for guaranteeing the performance of feedback control systems. MAVs use low-cost MEMS sensors which tend to be inaccurate, drift and fail. The unreliable output feedback resulting from the sensor uncertainties may cause performance deterioration and instability.

MAVs operate at low velocities in low Reynolds number aerodynamic regimes, have small mass and moments of inertia, exhibit complex nonlinear dynamics and are very susceptible to winds and gusts.

Manned air vehicles have well established handling and flying qualities criteria that can be used as design guide lines for control law design. Such criteria and design guidelines are not available for MAVs.

Thus, taking into account all the factors discussed above, innovative methods are needed for the control, guidance and navigation of MAVs. Classical PID controllers work well on MAVs. However, they require tuning for each MAV, and quickly loose performance in the presence of actuator failures or changes in MAV dynamics. In the robust control algorithms the deviations between nominal and true plant dynamics could actively be specified in uncertainty models leading to control designs accounting for those deviations. Sometimes, robust control is augmented by gain scheduling. However, even with the benefit of gain scheduling, robust control is unable to provide performance guarantees under conditions outside the pre-configured set of parameters.

Adaptive control in contrast to robust control does not assume an interval for the unknown plant parameters but treats them as unknown and tries to either determine the parameters in order to compute suitable controller gains or to directly estimate control gains.

The attractiveness of automatically adjusting a control law to unknown dynamics has motivated engineers to use a variety of adaptive control techniques, including neural networks, least squares estimation, and Lyapunov based methods for flight vehicles. Because the autopilots on MAVs are small, many of the adaptive control algorithms like those

employing least squares estimation may demand significant memory and computing power. However, Lyapunov-based Model Reference Adaptive Control (MRAC) methods are both simple and efficient, and are well suited for MAVs.

In the following sections, the salient features of the conventional MRAC methods, and two recent and significant variants of these methods are discussed briefly. These methods are applied to the roll control of a MAV whose data is available in open literature. The Sliding Mode Control, which can be considered as a robust adaptive control method, is also implemented for the sake of comparison. The conclusions drawn and plans for future work are also discussed briefly.

## 2. CONVENTIONAL MRAC METHODS

Model Reference Adaptive Control (MRAC) has been extensively applied for various classes of uncertain systems. The main objective of MRAC is to design a control signal such that the output of the plant tracks the output of a reference model.

There are two basic architectures of MRAC – Direct MRAC and the Indirect or Predictor-based MRAC [1]. In the direct architecture, the controller parameters are directly updated. In the indirect architecture, the plant parameters are estimated and used in the feedback law.

### Direct MRAC:

A block diagram of Direct MRAC architecture is shown in Figure 1. The direct MRAC comprises of:

*Plant (System):*

$$\dot{x}(t) = Ax(t) + bu(t), \quad x(0) = x_0$$

$$y(t) = c^T x(t)$$

where  $A$  and  $b$  are unknown constant parameters with known sign of  $b$ .

The plant dynamics can be rewritten as

$$\dot{x}(t) = A_m x(t) + b(u(t) + k_x^T x(t)), \quad x(0) = x_0$$

where  $A_m = A - bk_x^T$ .

*Reference Model:*

$$\dot{x}_m(t) = A_m x_m(t) + bk_g r(t), \quad x_m(0) = x_0$$

$$y_m(t) = c^T x_m(t)$$

where  $A_m > 0$  is chosen to meet the performance requirements, and  $r(t)$  is an external command.

*Adaptive Controller*

$$u(t) = -\hat{k}_x^T(t)x(t) + k_g r(t)$$

where  $k_g = \frac{1}{c^T A_m^{-1} b}$

*Adaptive Laws*

$$\dot{\hat{k}}_x(t) = -\Gamma x(t)e^T(t)Pb, \quad \hat{k}_x(0) = k_{x0}$$

where  $\Gamma > 0$  is the adaptation gain and  $P = P^T > 0$  solves the algebraic Lyapunov equation  $A_m^T P + PA_m = -Q$  for arbitrary  $Q = Q^T > 0$ , and  $e(t) = x(t) - x_m(t)$  is the tracking error.

### Indirect MRAC:

The architecture of Indirect MRAC is shown in Figure 2, and it comprises of the same plant and adaptive controller as in the case of Direct MRAC. But, a state predictor is used instead of a reference model, and has a different adaptive law.

*State Predictor:*

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b(u(t) + \hat{k}_x^T x(t)), \quad \hat{x}(0) = x_0$$

$$\hat{y}(t) = c^T \hat{x}(t)$$

*Adaptive Law:*

$$\dot{\hat{k}}_x(t) = -\Gamma x(t)\tilde{x}^T Pb$$

where  $\tilde{x} = \hat{x}(t) - x(t)$  is the tracking error.

The basic MRAC methods have good asymptotic response, but the transient response tends to be oscillatory and degrades further with the increase in the adaptive gain ( $\Gamma$ ). Hence, the basic MRAC methods are not suitable for the control of agile MAVs.

A large number of modified MRAC methods have been suggested in the literature, mainly to improve the transient response. One of these variants known as the ‘‘Modified Reference Model MRAC’’ method is discussed in the next section.

## 3. MODIFIED REFERENCE MODEL MRAC

The architecture of the Modified Reference Model MRAC (M-MRAC) method [2] is shown in

Figure 3. This is a simple and elegant modification of the Direct MRAC architecture.

The M-MRAC method is based on the fact that the initial large error in the control gains generates large transient excursions both in system's control and output signals. The output excursions in turn generate oscillations in the control signal. So, if the reference model is driven towards the system proportional to the tracking error, the system's effort to aggressively track the reference model can be prevented and the oscillations in the system's signals can be reduced. The reference model is modified as shown below.

*Modified Reference Model:*

$$\dot{x}_m(t) = A_m x_m(t) + b k_g r(t) + \lambda e, \quad x_m(0) = x_0$$

$$y_m(t) = c^T x_m(t)$$

where  $e = x(t) - x_m(t)$ , and  $\lambda$  is a design parameter.

The method provides guidelines for choosing the error feedback gain ( $\lambda$ ) and the adaptive gain ( $\Gamma$ ) together to achieve good asymptotic and transient responses with bounded errors.

#### 4. L<sub>1</sub> ADAPTIVE CONTROL

The architecture of the L<sub>1</sub> adaptive controller [1] is shown in Figure 4. The control signal  $u(t)$  is low-pass filtered to prevent high-frequency oscillations in system's signals. This simple innovation gives rise to a entirely new paradigm in adaptive control. The L<sub>1</sub> adaptive control architecture comprises of the following blocks:

*Plant (System):*

$$\dot{x}(t) = A_m x(t) + b(u_{ad}(t) + \theta^T x(t)), \quad x(0) = x_0$$

$$y(t) = c^T x(t)$$

where  $A_m = A - b k_m^T$  is Hurwitz,  $b$  and  $c$  are known constant vectors,  $u_{ad}$  is the adaptive component of the control input, and  $\theta$  is the unknown parameter.

*State Predictor:*

$$\dot{\hat{x}}(t) = A_m \hat{x}(t) + b(u_{ad}(t) + \hat{\theta}^T x(t)), \quad \hat{x}(0) = x_0$$

$$\hat{y}(t) = c^T \hat{x}(t)$$

where,  $\hat{\theta}$  is the estimate of the parameter  $\theta$ .

*Adaptive Law:*

The estimate of the parameter  $\theta$  is governed by the following projection-type adaptive law:

$$\dot{\hat{\theta}}(t) = \Gamma \text{Proj}(\hat{\theta}(t), -\tilde{x}^T(t) P b x(t))$$

*Control Law:*

$$u(t) = u_m(t) + u_{ad}(t)$$

where  $u_m(t) = -k_m^T x(t)$  is the static feedback component. The Laplace transform of the adaptive component of the control signal is given by

$$u_{ad}(s) = -C(s)(\hat{\eta}(s) - k_g r(s))$$

where  $r(s)$  and  $\hat{\eta}(s)$  are the Laplace transforms of  $r(t)$  and  $\hat{\eta}(t) = \hat{\theta}^T(t)x(t)$ , respectively,  $k_g = -1/(c^T A_m^{-1} b)$ , and  $C(s)$  is a stable and strictly proper low-pass filter transfer function with DC gain  $C(0) = 1$ .

Then L<sub>1</sub> adaptive controller must satisfy the following L<sub>1</sub>-norm condition:

$$\lambda = \|G(s)\|_{L_1} L < 1$$

where,  $G(s) = H(s)(1 - C(s))$ ,  $H(s) = (sI - A_m)^{-1} b$ ,  $L = \max_{\theta \in \Theta} \|\theta\|_1$ .

The schematic diagram shown in Figure 4 simply illustrates the concept of L<sub>1</sub> adaptive controller. In practice, a more rigorous implementation including matched and unmatched uncertainties, un-modeled dynamics, nonlinearities and actuator/sensor failures is used.

The main features of the L<sub>1</sub> adaptive controller, proven in theory and consistently verified in experiments using large UAVs as well as small MAVs can be summarized as follows:

- Separation (decoupling) between adaptation and robustness,
- Guaranteed robustness in the presence of fast adaptation,
- Guaranteed transient response for system's input and output, *without* resorting to persistency of excitation, high-gain feedback, gain scheduling of the controller parameters, and reconfiguration,
- Guaranteed (bounded-away-from-zero) time-delay margin,

- Uniform *scaled transient response* dependent on (admissible) changes in initial conditions, system uncertainty, and reference inputs,
- Performance limitations consistent with hardware limitations, and
- Suitable for development of theoretically justified Verification and Validation tools for adaptive feedback systems.

## 5. SLIDING MODE CONTROL

The Sliding Mode Control (SMC) method [3] may be considered as a special candidate of robust adaptive control methods, and is well suited for the control of small UAVs and MAVs. The utility of SMC is illustrated by applying the method for the pitch rate control of a typical small UAV.

The longitudinal short-period motion of the UAV can be described by the 2 DOF equations:

$$\dot{\alpha} = Z_{\alpha 0} + Z_{\alpha} \alpha + (Z_q + 1)q + Z_{\delta_e} \delta_e$$

$$\dot{q} = M_{\alpha 0} + M_{\alpha} \alpha + M_q q + M_{\delta_e} \delta_e$$

The pitching moment ( $M$ ) and  $Z$ -force coefficients are nonlinear functions of angle of attack ( $\alpha$ ). The pitch axis static stability is determined by the derivative  $M_{\alpha}$ . The sign of  $M_{\alpha}$  becomes positive when the UAV becomes statically unstable.

To design the sliding mode controller, let the sliding surface be defined as:

$$S = \tilde{q} + \lambda \int_0^t \tilde{q} d\tau$$

where  $\tilde{q} = q - q_d$  is the state error,  $q_d$  is the desired trajectory and  $\lambda$  is a gain. The first term in eqn. (x) represents the proportional feedback. The second term representing the integral feedback provides additional flexibility for a robust design.

The Sliding Mode Control comprises of two modes. In the first mode, which may be called as a reaching mode, the states beginning from arbitrary initial state are attracted towards the sliding surface  $S = 0$ . In the second mode, the states slide along the sliding surface  $S = 0$ . Thus, the state error  $\tilde{q}$  always converges to zero because  $S = 0$ . In the presence of uncertainty, discontinuous control law is used for accomplishing sliding motion.

Essentially, the sliding-mode control is based on the Lyapunov stability requirement:

$$V = \frac{1}{2} S^2 \quad \text{and} \quad \dot{V} = S \dot{S} < 0.$$

The time derivative of the sliding surface is given by

$$\dot{S} = \dot{\tilde{q}} + \lambda \tilde{q} = -\dot{q}_d + (M_{\alpha} \alpha + M_q q + M_{\delta_e} \delta_e) + \lambda \tilde{q}$$

Thus, the equivalent control to satisfy  $\dot{S} = 0$  on the sliding surface is given by

$$\delta_{e\_e} = \frac{1}{M_{\delta_e}} (\dot{q}_d - \lambda \tilde{q} - M_{\alpha} \alpha - M_q q)$$

The equivalent control ensures that the state trajectory remains on the sliding surface  $S = 0$  under ideal conditions. When the state is outside the sliding surface, a switching control ( $\delta_{e\_s}$ ) is applied to drive the system state trajectory to the switching surface. Thus, the SMC has the variable control structure:

$$\delta_e = \delta_{e\_e} - \delta_{e\_s}, \quad \text{where} \quad \delta_{e\_s} = K \text{sign}(S), \quad K > 0.$$

Usually, the sign function is replaced by the saturation function to avoid chattering. Thus,

$\delta_{e\_s} = K \text{sat}(S)$ , where the saturation function is defined as

$$\text{sat}(S) = \begin{cases} 1 & \text{if } S > \varepsilon \\ -1 & \text{if } S < -\varepsilon \\ S/\varepsilon & \text{if } -\varepsilon < S < \varepsilon \end{cases}$$

where  $\varepsilon$  is the width of a thin boundary layer surrounding the sliding surface. The results from the implementation of SMC for pitch rate control of MAVs are discussed in the next section.

## 6. APPLICATION TO MAVs

The adaptive control methods discussed in this paper were applied to the roll control of a small UAV called as "Ultra Stick UAV" [4]. The control objective is very simple. The roll attitude response of the UAV must track the response of a reference model

$$G_m(s) = \frac{6.612}{s^2 + 4.371s + 6.612}$$

for a wide range of uncertainty in the plant matrix  $A$ . A linear model of the Ultra Stick UAV was used for designing the controllers [4].

Figure 5 shows the roll response obtained using Direct MRAC controller. Similar response was obtained in the case of the predictor-based MRAC controller also. It can be seen from Figure 5, that the

transient response is oscillatory although the asymptotic response is satisfactory.

Figure 6 shows the roll response obtained in the case of M-MRAC controller. It can be noted that the oscillations are absent in the transient response and the asymptotic response is also satisfactory. Similar response was obtained in the case of  $L_1$  adaptive controller. A simple low-pass filter  $C(s)=1/s$  was used. The responses shown in Figs. 5-6 are for linear plant dynamics at a trim point.

Figure 7 shows the schematic for the implementation of sliding mode control for the pitch rate tracking of the Ultra Stick UAV. The following linear model of the UAV was used:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} = \begin{bmatrix} -7.8040 & 0.9013 \\ -141.27 & -35.203 \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} -0.2353 \\ -106.00 \end{bmatrix} [\delta_e]$$

The results from the SMC implementation are shown in Figs. 8 and 9. Although the plant dynamics are unstable the pitch rate tracking is very good.

## 6. CONCLUSIONS AND FUTURE WORK

The conventional MRAC, the modified reference model MRAC, and the  $L_1$  Adaptive control methods are briefly discussed in this paper. These four methods were applied to the roll attitude control of a small UAV called as Ultra Stick UAV. The M-MRAC and the  $L_1$  Adaptive control are very promising for the control of MAVs and small UAVs. The present study was carried out using linear models of a MAV. Simulations using 6 DOF models will be carried out to evaluate the robustness of M-MRAC and  $L_1$  Adaptive control methods in the case of model uncertainties, actuator failures and external disturbances.

The Sliding Mode Control is also found to be very suitable for MAVs which are highly agile and nonlinear in dynamics. An in-depth study of SM for the control of MAVs will be pursued further.

## 7. ACKNOWLEDGEMENTS

The authors wish to acknowledge the help received from Prof. Naira Hovakimyan and her group in providing valuable inputs on the  $L_1$  adaptive control scheme.

## REFERENCES

1. Evgeny Kharisov, Naira Hovakimyan, and Karl J. Astrom, "Comparison of Several Adaptive Controllers According to Their Robustness Metrics", AIAA 2010-8047,

AIAA G.N & C Conf., 2-5 August 2010, Toronto, Canada.

2. Vahram Stepanyan and Kalmanje Krishnakumar, "MRAC Revisited: Guaranteed Performance with Reference Model Modification", 2010 American Control Conference.
3. K. David Young, "A Control Engineer's Guide to Sliding Mode Control", IEEE Trnas. on Control System Technology, Vol.7, No.3, May 1999.
4. Yew Chai Paw, "Synthesis and Validation of Flight Control for UAV", Ph.D. Thesis, University of Minnesota, December 2009.

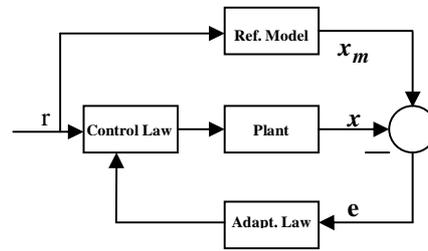


Figure 1. Direct MRAC Architecture

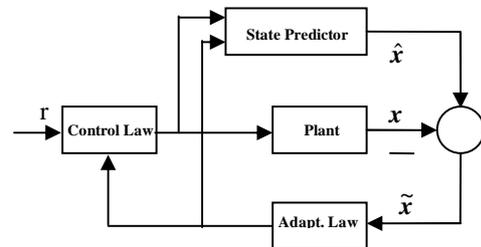


Figure 2. Indirect MRAC Architecture

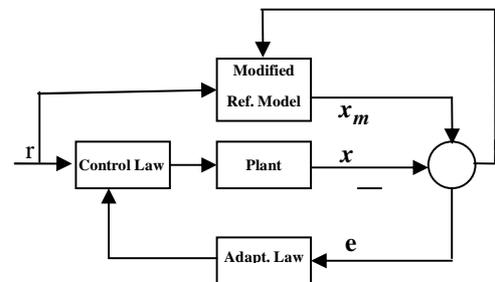


Figure 3. Modified Reference Model MRAC

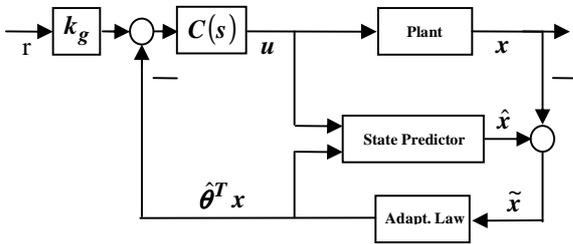


Figure 4.  $L_1$  Adaptive Control Architecture

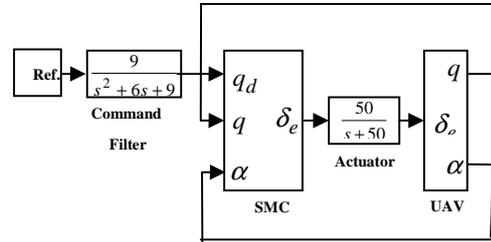


Figure 7. Sliding Mode Control implementation

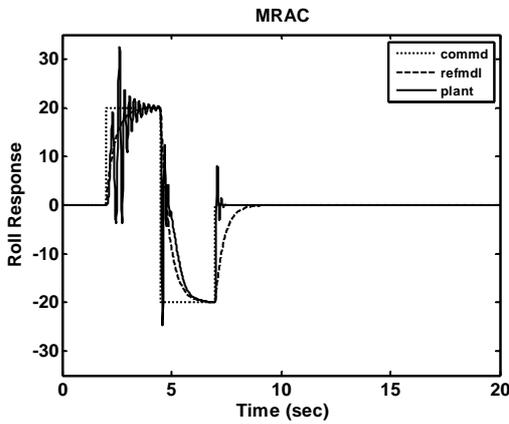


Figure 5. Roll response using MRAC control

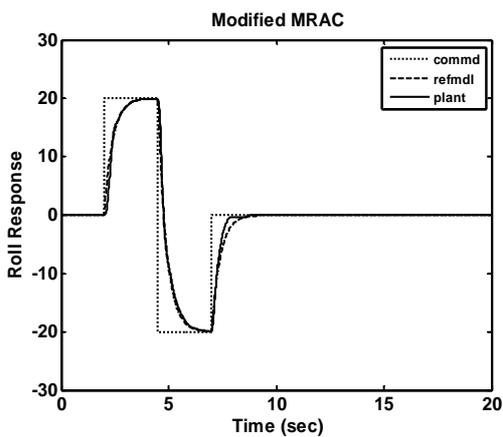


Figure 6. Roll response using M-MRAC control

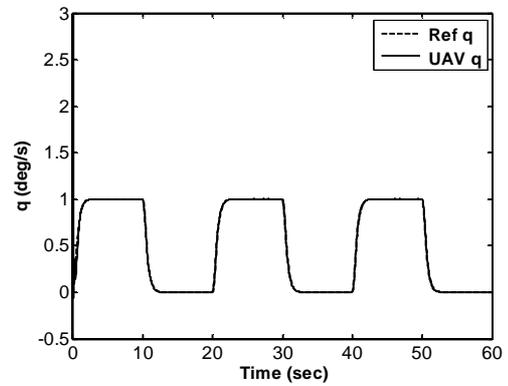


Figure 8. UAV pitch rate tracking

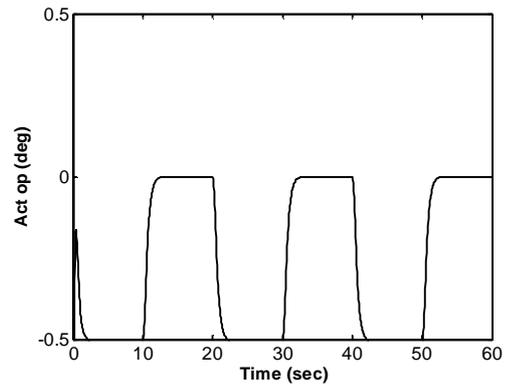


Figure 9. Actuator output for UAV pitch rate track