Form Error Evaluation by Rapid Registration Method

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Abstract:
The automated inspection to evaluate form error on machined parts is an area of active research. In this paper we develop a methodology for inspecting form error based on Least Square Plane Fitting and Modified Iterative Closest Point (MICP) algorithm to quantify the manufacturing errors. In the current paper we use analytically defined surfaces to validate our algorithm so that the additional error introduced due to tessellated representation of the geometry is eliminated, which is the case for arbitrary machined parts where the CAD geometry is represented by facets. The rigid registration carried out on inspected set of points with the nominal CAD points is described in two parts. The first part of the study deals with establishing manufacturing errors by generating inspection data in a known reference system called hard inspection on a Coordinate Measuring Machine (CMM), referred to as hard inspection, while the second part of the study deals with capturing the errors by generating inspection data from an unknown reference system called soft inspection. Both the results are evaluated and verified with experimental case studies. The results confirm the feasibility of the methodology and show that the algorithm is effective and accurate and makes this method attractive to carryout automated inspection of machined components.

Keywords: Soft inspection, hard inspection, soft registration, hard registration, online inspection, offline inspection.

Nomenclature:
- \[ \mathbf{M}_i \] Nominal Part coordinate (3x1)
- \( m_1, m_2, m_3 \) and \( m_4 \) Four closest points in nominal part point dataset \( M \)
- \( [q_0, q_1, q_2, q_3] \) 4D vector of quaternions
- \( \mathbf{P}_i \) Part Inspection coordinate
- \( R \) Rotation Matrix
- \( T \) Translation vector
- \( \{ \mathbf{v}_i \} \) Vectors of \( \{ \mathbf{v}_1 \} \) and \( \{ \mathbf{v}_2 \} \) respectively (n x 1)

1 Introduction

The industrial products today demand large number of functional features involving close tolerances to derive high levels of performance. This is becoming the order of the day in many sectors such as aerospace, automobile and white goods to name a few etc. with a marked increase in utilization of Class A surfaces. These feature and function rich components have made the inspection a very complex process given the requirement of measuring close tolerances at a fast rate with reliable results. The need to provide the tolerance on dimensions defining the part in a drawing is due to the inherent inability of the manufacturing processes to achieve the dimensions dictated in the drawings or in the CAD model, and the essence of manufacturing is to machine a part within the defined tolerance limits. Deviations which affect the form and dimensions of the part, could be due to many reasons such as multiple operations and processes involved in achieving the desired dimensions on the finished part, inbuilt machine tool errors such as run out, position, perpendicularity, circularity among others, apart from human errors. The acceptance of a part demands the conformance of dimensions to be within the allowable tolerances specified by standards and it is here that the inspection plays a crucial role in deciding whether the part meets the agreed acceptance criteria or not. One of the objectives of manufacturing is also to carryout automated inspection so as to reduce human interference and achieve reliable and consistent quality with the manufactured part satisfying the design intent. With the ever-increasing demand to make inspection cost and time effective, it is becoming necessary to automate the inspection procedure. In this paper we have adopted a combination of online and
offline inspection processes, wherein the online inspection stage captures the data and the offline inspection stage carries out the data analysis and reporting, thus relieving the CMM to perform unhindered inspection thereby improving its throughput and productivity. It is necessary to note that the CAD model could be represented by a set of curves such as B-Splines or NURBS to define a wire frame geometry or by any of the modern surface representation techniques using a set of points to define the actual geometry, and any of these methods in principle would introduce a geometrical error compared to an analytically defined surface. However, within the engineering branch of knowledge, we know that the geometric error could be reduced to any predefined levels of accuracy at the expense of computational effort and resource requirements. In this paper an effort is made to generate analytically definable objects wherein the basic geometric error is nullified along with the associated computational effort. The geometric integrity facilitates measurement of the manufacturing error using the hard inspection route by application of CMM and comparing the results obtained by soft inspection route through the application of Modified Iterative Closest Point (MIBC) algorithm. This paper proposes to present the results on various cases involving application of least square plane fit for inspection on planar features (flatness parameters) and Modified ICP on analytically defined objects. In each of these cases the unordered inspection datasets are input into our algorithm and the manufacturing deviations are captured. The components were inspected on CMM, which is the defacto industry standard, whose tactile method of measurement eliminates the deficiencies caused by the presence of outliers. Further, the ability of the CMM to measure an almost endless variety of geometrically complex components in a rapid and accurate way has led to their widespread use in industry. The hard inspection results obtained from CMM are compared against the soft inspection procedure adopted in this paper and the results show good concurrence. The method is experimentally verified explicitly on planar, cylindrical, toroidal and spherical objects defined analytically.

Establishment of proper correspondence between the inspection coordinate system used for measurement in CMM and that in the design coordinate system used for the part is necessary to determine the manufacturing error and to verify its conformity to the tolerances specified on the machined part in the drawing. Presently, this correspondence is established a priori through the use of high-precision markers and custom made jigs and industry practices often adopt high precision fixture to locate the part orientation [2]. This paper employs the Modified Iterative Closest Point (MICP) method [3] to determine the transformation relating the measured and nominal point set, as the registration of the two point sets enables establishing machining deviations on the part with respect to its nominal geometry with the error distance defining the manufacturing error. The paper also evaluates the application of Least Square Plane (LSP) fit method for finding the deviation of parts having planar surfaces with respect to its nominal form. The discrete measurement data of a planar surface is obtained from CMM and the rigid surface is fit to obtain an optimal solution. The optimal solution is compared with the dimensional results that are obtained from hard inspection process. The LSP fit method was implemented on parallel bar and the inspection results are found to be very encouraging.

The reminder of this paper is organized as follows: Section 2 reviews the current state of the art in the registration of point datasets. An overview of the rapid registration technique employed is described in Section 3, bringing out the importance of automatic registration of the inspection dataset with the nominal data reference system. Section 4 describes the registration procedure adopted in detail. Section 5 presents the implementation and results of the proposed approach to actual machined components. Conclusions and suggestions for further work are presented in Section 6.

2. Literature survey

The application of metrology equipments such as Advanced CMM's, Laser Trackers, Scanners with articulated arms etc. to carry out rapid and automatic inspection of complex components and assemblies is a recent approach and a subject of active research. The last decade has seen several efforts in the areas of registration of 3D point cloud data. These efforts have been primarily restricted to the areas of computer vision, image processing, pattern recognition, reverse engineering, texture mapping and so on.

The most popular approach to solving the registration problem is the class of algorithms based on the Iterative Closest Point (ICP) technique for pattern matching between point set A and point set B suggested by Besl and Mckay [4]. This algorithm is a general purpose registration method for freeform curves and surfaces wherein it is assumed that the data point set is a subset of the model point set and has seen many implementations over the years and by far provides the most accurate registration results [2]. The ICP has three basic steps:
1. Pair each point of the object dataset to the closest point in the model dataset.
2. Compute the motion that minimizes the mean square error between the paired point sets.
3. Apply transformation to the object dataset and update the mean square error.

Zhang et al [5] highlight on the importance to consider the source of error, which is related to the design of the
3D scanner and the alignment between the inspection point cloud and the nominal CAD data. This is subdivided into error due to registration of multiple point clouds into one common coordinate frame and registration of the entire point cloud to the CAD model. Rusinkiewicz and Levoy [6] demonstrate an implementation of ICP where a rough initial alignment is always available and focuses on aligning a single pair of mesh. The error metric used here is root mean square point to point distance between the corresponding points in the two meshes. Lukacs et al [7] address a problem arising in the reverse engineering of solid models from depth maps and present a set of methods for the least squares fitting of spheres, cylinders, cones and tori to three dimensional datasets to identify and fit surfaces of known type, wherever there is a good fit. Meng et al [8] put forward a statistical sampling plan to determine a suitable sample size to define the geometry accurately with sufficient confidence. Narayanan et al. [9] proposes $L_2$ polynomial approximation for evaluation of form errors of engineering surfaces with applications involving straightness, circularity, flatness and cylindricity. Roy [10] carries out tolerance analysis for cylindrical surfaces based on computational geometry techniques and numerical analysis methods for computer aided inspection based on least squares. Shi and Xi [11] carry out registration from a tessellated representation of the surface. As in inspection, the data from the inspection device is a set of points, the dataset with which comparison needs to be done has to be either a smooth representation of geometry or a sampling of points from it. A tessellated representation of a surface cannot be used in all cases (including complex parts), as the inbuilt tessellation error is not accounted for in these approaches. Nevertheless, this methodology is more suited to image-based registration applications rather than to the area of computational metrology. Meng, Yau and Lai [12] present a method capable of determining actual measured points by minimizing the sum of the squared distances of the measurement data from the surface of the part with respect to parameters of a rigid body transformation where the transformation matrix is determined by least square method. The paper states that when the part deviation from its nominal position and orientation is small, the transformation is recovered rapidly, while incase of parts with large deviation where minimization problem is highly nonlinear, a method to estimate the region of convergence is to be found. Weber et al [13], introduces a unified linear approximation technique to evaluate the forms of straightness, flatness, circularity and cylindricity. Non-linear equation for each form is linearized using Taylor expansion and then solved as a linear program.

Gilbert et al [15], develops a methodology for inspecting the form of any type of smooth surface by collecting a sample of contact points and introduces a regression method derived from least square support vector regression that finds the deformation shape of such surfaces with respect to their nominal form. Low and Lastra [16], demonstrate that registration failures can occur for several reasons such as insufficient overlap between the two surfaces, very large range measurement errors, insufficient geometric constraints on the 3D rigid body transformation between the two surfaces and large difference in the initial relative pose between the two surfaces. However, it is well known that when the initial pose is closer, the ICP method provides a monotonical convergence to the local minima and this is not the case always in practical conditions. The methods described in the following section aims to address these limitations.

3. Automatic Inspection of Components

The traditional practice of carrying out inspection of complex parts is through the usage of templates or gauges that define the acceptable surface geometry in local regions of the part (templates for example are defined at various sections of the part). Gauges are usually in pairs (e.g. go-no go gauge) - one that checks if the dimension is within the prescribed lower limit of deviation and the other that checks if the dimension is within the higher limit of deviation, thus capturing the range of acceptable sizes. The advent of CMM has brought in two scenarios. In the first case, the part to be scanned by the CMM is positioned in a well-defined manner using external jigs, fixtures or additional features that are built into the part for the specific purpose of locating it. In this scenario, the reference frame with respect to which the inspection data is obtained is well-defined enabling easy comparison with the nominal data. We refer to this as hard registration of inspection data and the inspection process is termed as hard inspection.

An alternative to the above is to scan and obtain inspection data from the part when it has been placed on a CMM in an arbitrary location without the use of any fixtures or markers. In this case, the reference frame in which the inspection data has been obtained is to be matched with reference frame of nominal model. We refer to this as soft registration of inspection data and the inspection process is termed as soft inspection. The soft registration approach is clearly more desirable as it is lower in cost (no investments on complex and precision jigs and fixtures), flexible (any part that can be mounted on the CMM bed can be inspected) and quicker. An additional advantage of this process is that this process also eliminates the need for additional locational information such as tooling hole reference datum, position, perpendicularity and parallelism data of parts that have to be trimmed after inspection. Further as the data analysis and reporting activities are carried out offline; the CMM can perform unhindered inspection thus
improving its productivity. As these processes eliminate the need for much of the human intervention as required in traditional inspection, the whole method lends itself to a high level of automation.

The first part of the study deals with evaluation of flatness adopting the Least Square Plane fit approach. The LSP fit assists in capturing the flatness of a planar surface and also define the parallelism between the two given planar surfaces. The scanned data points from CMM are used to fit the least square plane. The results obtained are validated by hard inspection and compared against the LSP fit method. The results show good degree of confidence any difference in results that may occur could be attributed to different measurement techniques, algorithms, measuring environment, uncertainties due to calibration or human error.

The second part of the study deals with evaluating the form errors on analytically defined surfaces and to carryout its inspection based on the modified iterative closest point algorithm as described in [3]. A variety of algorithms has been developed to evaluate the form error on parts. Though the least square method is the commonly used method in which the sum of the squares of the deviation from the nominal surface is minimized, it does not provide the minimum zone result and often overestimates the tolerance zone resulting in the rejection of parts that are within the tolerance specifications. Even though a variety of techniques have been developed which improve upon the least square method, many of which provide evaluation against the minimum tolerance zone, these are mathematically complex and often computationally slow for cases where a large number of data points is to be evaluated [13]. Therefore, we employ MiP algorithm to evaluate the form error on analytically defined surfaces viz., cylinder, sphere and tori. These surfaces are machined and the manufactured parts go through the procedure of hard and soft inspection, and the results present close convergence.

4. Registration methodology

The primary parameters that an ideal registration algorithm needs to satisfy [17] are: the algorithm should be fast, has to be accurate, where after successful registration, the distance between corresponding points in the region of interest is less than 0.1mm, which is the well-known Target Registration Error (TRE) parameter introduced by Maurer et al. [18][21]. In addition the algorithm has to be robust, automatic and reliable. This section outlines the basic concepts of least square plane fitting adopted for evaluating the form error for planar surfaces and then details the implementation of modified iterative closest point for registration of analytical surfaces.

4.1. Least Square Plane (LSP) fit

Given a set of points $P \in \mathbb{R}^n$, where $n=3$ and $P = \{P_i\}$ for $i = 1, ..., N$ representing the cartesian coordinate measurements.

Given the equation of a plane as $ax + by + cz = d$.

Let $r_i$ be the distance of point $i$ from the plane and

$$r_i = ax_i + by_i + cz_i - d = 0 \quad (1)$$

with

$$r_i = (ax_i + by_i + cz_i - d) / \sqrt{a^2 + b^2 + c^2} \quad (2)$$

Given a set of $N$ points (a point $i$ has coordinate $x, y, z$).

we wish to minimize $Q$,

$$Q = \sum_{i=1}^{N} r_i^2 \quad (3)$$

From (3)

$$\frac{dQ}{da} = \sum_{i=1}^{N} \left[2x_i (ax_i + by_i + cz_i - d) \right] = 0 \quad (4)$$

$$\frac{dQ}{db} = \sum_{i=1}^{N} \left[2y_i (ax_i + by_i + cz_i - d) \right] = 0 \quad (5)$$

$$\frac{dQ}{dc} = \sum_{i=1}^{N} \left[2z_i (ax_i + by_i + cz_i - d) \right] = 0 \quad (6)$$

$$\frac{dQ}{dd} = \sum_{i=1}^{N} \left[-2(ax_i + by_i + cz_i - d) \right] = 0 \quad (7)$$

Equation (7), noting that $\Sigma(d) = N$ can be rewritten as

$$d = ax_o + by_o + cz_o \quad (8)$$

where

$$x_o = \Sigma x/N, \quad y_o = \Sigma y/N, \quad z_o = \Sigma z/N$$

Equation (8) shows that the best fit plane passes through the centre of mass. Subtracting the centre of mass from
each point and substituting into equations 4 - 6, we get a set of simultaneous equations which can be written as:

\[ WP = 0 \]  

\[ W = \begin{bmatrix} \sum(x_i - x_p)^2 \sum(y_i - y_p)(z_i - z_p) \\ \sum(x_i - x_p)(y_i - y_p) \sum(y_i - y_p)(z_i - z_p) \\ \sum(x_i - x_p)(z_i - z_p) \sum(y_i - y_p)(z_i - z_p) \end{bmatrix} \]

We need to solve equation (9) to obtain LSP.

Equation (9) can always have a trivial solution for \( a = b = c = 0 \).

To avoid this trivial solution we need to impose conditions on the coefficients of the plane. The most common condition being:

\[ a^2 + b^2 + c^2 = 1 \]  

With the condition (10), solution to (9) becomes an eigenvalue problem where

\[ WE = VE \]  

Where \( V = \text{eigenvalue} \) and \( E = \text{eigenvector} \) and equation (11) will return three eigenvalues and 3 x 3 eigenvector.

The eigenvalues will be the sum of the squares of the distances and the eigenvector will provide the three sets (a set is a column of the matrix \( E \)) of \( a, b, c \) values.

One then needs to choose the set \( a, b \) and \( c \) associated with the smallest of the eigenvalues. It should be noted that:

(a) The three sets of \( a, b \) and \( c \) contained in \( E \) are orthogonal to each other and represent the best, intermediate and worst planes respectively.

(b) If the eigenvalues are similar or same within the margin of error, this suggests that the data coordinates are either degenerate or symmetric.

In summary, to calculate LSP we need to get the eigenvalue and eigenvectors of \( W \).

4.2. Modified Iterative Closest Point algorithm

The registration procedure adopted in this paper is based on the well known Iterative Closest Point (ICP) algorithm [4]. The ICP algorithm is based on determining the correspondence between points in two datasets. In the present approach we use the measured points from a Coordinate Measuring machine (CMM) to represent the inspection point dataset. The nominal point dataset specification of the part is available from the analytical geometry and is used for both registration and comparison and this allows the inspection task to be performed independent of CAD system. The schema for automated inspection is shown in fig. 1 where the part inspection data set is obtained in the Inspection Coordinate System (ICS) and the nominal dataset is obtained in the Model Coordinate System (MCS) [3].

[Diagram of registration process]

Fig. 1 - Schema for automated component inspection

The registration procedure consists of establishing a matching between two sets of three dimensional point data sets called the Part inspection dataset \( P \) and the Nominal point data set \( M \), where their point elements are defined as

\[ P = \{ \hat{P}_i \} \] for \( i = 1, \ldots N_p \) and \( M = \{ M_i \} \) for \( i = 1, \ldots N_m \)

where \( N_p \) and \( N_m \) are the point set sizes.

The spatial translation between two sets is a linear transformation vector in \( \mathbb{R}^3 \) and is the difference in the location of centre of gravity of both the sets. For the rotational alignment of the two point sets we use quaternion algebra [19].

Here, the quaternion is a 4D vector denoted as \( q = [q_0, q_1, q_2, q_3] \) and is used to represent the 3D rotation which is of practical importance to us. The norm of a quaternion \( N (q) \) is conventionally the sum of the squares of the four components. The 3x3 rotation matrix \( R \) generated by a unit quaternion is given as
Therefore, the coordinate transformation which involves the rotation matrix \( R \) and the translation vector \( T \) from the part inspection coordinate system to the model coordinate system is solved using the quaternion algebra, where

\[
M_i = R_i P + T_i
\]

Here, \( M_i \) denotes the coordinates in the model frame and \( P \) denotes the coordinates in the part inspection frame. A necessary condition for the current algorithm is that the inspection data set is assumed to be a small subset of the nominal point set which is represented by a large point set within the limits of available computer resources and acceptable accuracy to achieve the desired tolerance.

The process of modified ICP can be classified in two stages.

i) For every point in \( \{ P_i \} \ i \in [1, N_p] \), the closest point \( \{ M_i \}, i \in [1, N_p] \) in the Nominal Point set is found.

A fast iterative method is adopted for establishing the correspondence with the closest points and when the component region have large curvature, the input nominal point set need to be dense so as to define the geometry accurately.

ii) The transformation as given in equation (2) above is applied for every point in the inspection points set \( P_k \). The transformed inspection point set at say the \( j^{th} \) step of iteration is now closer to its corresponding points \( M \) in nominal point dataset. The mean squared objective function to be minimized is the distance function given by:

\[
d_k = \frac{1}{m} \sum_{k=1}^{m} \| T_k P_k - M_k \|^2
\]

The process enumerated in iterative steps (i) and (ii) is repeated using the updated inspection point set \( P_k \) until \( d_k \) converges to a predefined threshold tolerance [20] or when a predetermined number of iterations is reached.

In representing the nominal point data set in the current work, the point data set is generated as a topologically uniform rectangular grid in the parametric space (\( u, v \)). The least square is calculated between points in correspondence. The objective function minimized is the sum of squared distances divided by the number of points in inspection data set. Since the points in the nominal point data are sampled, the actual error need not be the distance between the points in correspondence or the average of the square of this error. The actual error that is of interest is the normal distance from part inspection set to the nominal surface which is obviously not captured in the procedures adopted till now. We define the error as the distance between the inspection point and the surface in the vicinity of its corresponding point in the nominal point data set. The vicinity is defined by facets between four points in close correspondence with the inspection point. The procedure is illustrated in the fig. 2 below.

Here, \( M_1, M_2, M_3 \) and \( M_4 \) are the four closest points in the Nominal Point data set \( M \) to the part inspection point \( P \) which can be represented as four triangles in 3D each of which will define an analytical plane. The localized region based approach captures the normal distance of \( P \) to each of these triangles, the minimum of which establishes the closest deviation of the manufactured component to its native analytical geometry. This modified approach eliminates the possibility of two inspection points having the same closest point from the geometry set. It should be observed that the least square norm to define the error acceptability criteria fictiously reduces the error to very low values. This becomes more pronounced when there are very limited numbers of outliers against a large number of very close matching points. This will lead to erroneously accepting inspection points which otherwise are far away from the acceptability criteria. The local quadrilateral facet approach provides a practical technique in finding the absolute distance error as the least squares method does not necessarily define the accurate part tolerance deviation as explained above. Computing the distance between the inspection point and the tangent plane at its corresponding point will also not be the right measure for the same reason that the corresponding point is obtained from a sample and therefore may not be the actual closest point on the surface. Tangent plane at this point may therefore not be the correct approximation of the surface around the closest point. This point wise distance could be well above the acceptable tolerance, whereas the least square distance may be well below the permitted tolerance in the iterative scheme thereby leading to a wrong conclusion of accepting the erroneous (out of tolerance) component. Thus it is more appropriate to use the point wise distance criterion to establish the manufacturing deviation rather than the least square convergence.
5. Implementation and results

The algorithm based on Least Square Plane fit and the Modified Iterative Closest Point was evaluated for establishing the form error by carrying out inspection on multiple cases of components defined by analytical geometry. These included a parallel bar, cylinder, sphere and tori. The results obtained by inspecting these components are described below. The results demonstrate the performance of the above methods in a real world inspection environment.

5.1 Inspection of Parallel Bar by LSP method

The inspection of the Parallel Bar was conducted on a CMM as shown in Fig. 3. The inspection points were captured randomly over the entire surface of the artifact to establish the flatness error. Flatness tolerance which is defined as zone between two parallel planes, within which a surface must lie, is a form tolerance which does not need to be related to a datum. The inspection data generated from the CMM for the parallel bar was applied to establish the error by the least square plane fit methodology as detailed in Section 4.1. The hard inspection result for the parallel bar was also obtained from the CMM. Both the inspection results showed that the error varied from -0.008 mm to 0.0015 mm, thus displaying a incredible convergence of the inspection results from soft inspection methodology followed in this paper with the hard inspection results carried out on CMM. Fig. 4 shows flatness error variation by the two methods to be in very good conformance.

5.2 Cylinder:

A cylinder of 40mm diameter was machined on a turning centre and the part was inspected on a CMM as shown in Fig. 5. The graph comparing the inspection results carried out both by hard and soft inspection is in Fig. 6. The hard inspection results show the machining error range from 24.4 micrometer to 35.6 micrometer while the soft inspection results show the machining error range from 1.3 micrometer to 34 micrometer thus showing an excellent conformance of the inspection method.

Fig. 3 Inspection of Parallel Bar on CMM

Fig. 4 Comparison of Hard and Soft inspection results for Planar surface

Fig. 5 Hard inspection of Machined Cylinder
5.3 Torus

A torus which is a surface of revolution and azimuthally symmetric about the z-axis, with major and minor radius of 27.5 mm and 10 mm, thus having an aspect ratio of 2.75 with toroidal and poloidal angles of 180° respectively was machined on Vertical CNC machining centre. The machined ring torus was inspected on a CMM as shown in fig 7. The graph comparing the inspection results carried out by hard and soft inspection is shown in fig 8. The hard inspection results show the machining error range from 5.7 micrometer to 38.4 micrometer while the soft inspection results show the machining error range from 3.4 micrometer to 41.2 micrometer thus showing an excellent conformance of the inspection method.

5.4 Sphere

A reference sphere of 25 mm diameter as shown in fig. 9 was taken for carrying out hard inspection on CMM. The hard inspection results showed that the error on the spherical artifact varied from 0 microns to a maximum of 6 micrometer while the soft inspection results showed the error variation from 0 to 9.83 micrometer, thus showing a very good conformance of results. Fig. 10 shows the comparison of results derived from the two methods.
6 Conclusions

A form error evaluation method for different surface profiles, which requires very little human interaction was proposed and evaluated. The tolerance specifications as per the standards were made applicable for the various analytically defined shapes. The inspection plan based on both online and offline methodology was followed with data registration carried out employing the LSP fit and modified ICP algorithm. The soft inspection carried out on the machined parts demonstrates their acceptability based on the tolerance specifications. The soft inspection results were verified against the hard inspection carried out using a CMM and the results show close conformance. Validation of the inspection procedure by using analytically definable objects have showed that the methodology adopted was indeed capable of inspecting objects of complex shapes represented by faceted geometry and the future work would be to improve the general applicability of the proposed approach. Further, it is well documented that when the initial pose is closer, the K'P algorithm provides monoronical convergence to the local minima. The method being sensitive to initial orientation, we propose to address this issue by the usage of a spherical gauge to cater to inspection conditions where the ICS and MCS are far off, and this investigation would be part of our upcoming research activities.

Appendix A. Standard Mathematical definition of tolerance zones [14]:

(i) Flatness. Flatness is the condition of a surface having all elements in one plane. A flatness tolerance specifies a tolerance zone defined by two parallel planes between which the surface must lie. A flatness zone is a volume consisting of all points \( \vec{P} \) satisfying the condition:

\[ |\hat{T} \cdot (\vec{P} - \vec{A})| \leq \frac{t}{2} \]

where
\( \hat{T} \) = the direction vector of the parallel planes defining the flatness zone
\( \vec{A} \) = a position vector locating the mid-plane of the size of the flatness zone
\( t \) = the size of the flatness zone (the separation of the parallel planes)

(ii) Cylindricity: A cylindricity tolerance specifies that all points of the surface must lie in some zone bounded by two coaxial cylinders whose radii differ by the specified tolerance. A cylindricity zone is a volume between two coaxial cylinders consisting of all points \( \vec{P} \) satisfying the condition:

\[ ||\hat{T} \times (\vec{P} - \vec{A})|| - r \leq \frac{t}{2} \]

where
\( \hat{T} \) = the direction vector of the cylindricity axis
\( \vec{A} \) = a position vector locating the cylindricity axis
\( r \) = the radial distance from the cylindricity axis to the center of the tolerance zone
\( t \) = the size of the cylindricity zone

(iii) Circularity (Roundness): Circularity is a condition of a surface where:
(a) for a feature other than a sphere, all points of the surface intersected by any plane perpendicular to an axis are equidistant from that axis;
(b) for a sphere, all points of the surface intersected by any plane passing through a common center are equidistant from that center.
A circularity tolerance specifies a tolerance zone bounded by two concentric circles within which each circular element of the surface must lie, and applies independently at any plane described in (a) and (b) above.
A circularity zone at a given cross-section is an annular area consisting of all points \( \vec{P} \) satisfying then conditions:

\[ \hat{T} \cdot (\vec{P} - \vec{A}) = 0 \]

and

\[ ||(\vec{P} - \vec{A})|| - r \leq \frac{t}{2} \]

where
\( \hat{T} \) = for a cylinder or cone, a unit vector that is tangent to the spine at \( \vec{A} \). For a sphere, \( \hat{T} \) is a unit vector that points radially in all directions from \( \vec{A} \)
\( \vec{A} \) = a position vector locating a point on the spine
\( r \) = a radial distance (which may vary between circular elements) from the spine to the center of the circularity zone \( t > 0 \) for all circular elements
\( t \) = the size of the circularity zone.

References:


