ON THE UPPER LIMIT OF THE OSWALD'S EFFICIENCY FACTOR FOR AN AIRPLANE

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Abstract

With a view to understand the maximum limit on the airplane Oswald's efficiency factor e_0 in the quadratic represent on of the Drag Polar, an analytical expression for eon has been derived in terms of the ratio of the horizontal tail lift-coefficient to wing lift coefficient. Maximising e_0 with respect to this ratio shows that $(e_0)_{max}$ is a strong function of the tail span to wing span ratio. It is observed that for reasonably high values of this span ratio $(e_0)_{max}$ could exceed values of unity.

Introduction

The parabolic drag polar for the wing alone (any lifting surface) is generally written as

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e} \qquad (1)$$

where A is the aspect ratio and $e = \frac{1}{1+\delta}$; δ is a positive quantity and depends on the spanwise load distribution on the wing and e is called the wing span efficiency factor. For elliptic distribution of cC_1, $\delta = 0$ and $e = 1$, which is the maximum value of e for a single lifting surface. However, for a complete aircraft, we have more than one lifting surface viz. wing, tail, canard etc. and further a fuselage which carries these surfaces together. The drag polar for the complete airplane is also written in a form similar to that of a lifting surface as

$$C_D = C_{D0} + \frac{C_L^2}{\pi A e_0} \qquad (2)$$

where A = Aspect ratio of wing (major lifting surface).

The quadratic term in C_L in Eq. (2) includes all the lift dependent drags - both viscous and inviscid. Here e_0 is defined as Airplane efficiency factor or Oswald's efficiency factor and is generally obtained by plotting the total drag-coefficient of the airplane - obtained through measurements or estimates - against the square of the lift coefficient and fitting a straight line for linear variation range. Its value is generally less than unity (Ref. 1) and most books on aircraft design recommend a value of e_0 in the range of 0.7 to 0.85 for preliminary design (Ref.2). The value of the e turns out to be generally less than unity because the lift distribution on the wing is nonelliptic, the contribution of the other lifting surfaces is comparatively smaller and the nonlifting components such as fuselage, nacelle, landing gear etc. force the lift distribution towards non-elliptic.

A study of the wind tunnel test results on some of the recent configurations has shown values of e_0 greater than unity. A cursory search in the literature on some configurations has shown values of $e_0 > 1$; e.g. for Learjet aircraft $e_0 = 1.1$ (Ref.3). These findings have shaken the common belief of many aerodynamics designers on the range of values of e_0 and at times have led to suspecting the correctness of the wind tunnel test results. With a view to resolve this issue, a survey was carried out by the authors in which they came across couple of references (Ref. 4-5) wherein it is mentioned that, though high values of e_0 are not common, they do occur and it is possible to have values of e_0 exceeding unity. In fact Ref.6 lists values of e_0 for typical airplanes, which include a few with e_0 as high as 1.1. In Ref. 5, while considering the estimation of camber drag the author addresses a situation where e_0 could be greater than or equal to 1. The survey carried out by the present authors has thus established the possibility of e_0 assuming values greater than unity. In this study the authors have made an attempt to prove analytically that e_0 for the complete aircraft configuration could be greater than unity.

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Analysis

The analysis is first presented for a configuration with two lifting surfaces which typically represents a conventional airplane configuration. Subsequently, it is generalised to a multi component configuration.

Consider a configuration consisting of a wing of area S and a horizontal tail of area S_T. The induced drag coefficients of the individual surfaces can be written as:

$$C_{DW} = K_W C_{LW}^2 ; \quad C_{DT}^* = K_T C_{LT}^{*2} \tag{3}$$

where C_{DW}, and C_{LW} are the induced-drag and lift coefficients respectively and are based on the wing area S. C_{DT}^* and C_{LT}^* are the tail induced-drag-coefficient and lift-coefficient respectively with reference to S_T. Defining C_{LT} and C_{DT} as the tail lift-coefficient and induced-drag-coefficient based on S, one can write :

$$C_L = C_{LW} + C_{LT} ; \quad C_{D\,i} = C_{DW} + C_{DT} \tag{4}$$

Defining an Equivalent induced drag factor K for the total configuration we can write

$$K = K_W \left[1 - \frac{C_{LT}}{C_L} \right]^2 + K_T \frac{S}{S_T} \left[\frac{C_{LT}}{C_L} \right]^2 \tag{5}$$

From Eq. (5) we can write an expression for the Oswald's efficiency factor e_0 of the airplane as

$$\frac{1}{e_0} = \frac{1}{e_w} (1 - x)^2 + \frac{1}{e_T} \left(\frac{b}{b_T} \right)^2 (x)^2 \tag{6}$$

where e_w and e_T are the planform efficiency factors of the wing and tail surfaces respectively and b and b_T are the wing and tail spans. x represents the ratio C_{LT}/C_L. Eq. (6) shows that e_0 *depends, besides on* e_w and e_T, on the ratios b/b_T and C_{LT}/C_L. It is however to be noted that this analysis has not taken the effects of fuselage into consideration. These effects are generally towards reducing the value of e_0 and hence the values of e_0 from the wind tunnel test results and present prediction may not exactly match.

Equation (6) shows that $1/e_0$ is quadratic in C_{LT}/C_L and will have a maximum value for a particular value of C_{LT}/C_L. It can easily be shown that this value of C_{LT}/C_L is given by

$$\left(\frac{C_{LT}}{C_L} \right)_{opt} = \frac{1}{1 + \frac{e_w}{e_T} \left(\frac{b}{b_T} \right)^2} \tag{7}$$

The corresponding maximum value of e_o is given by

$$(e_0)_{\max} = e_w \left[1 + \frac{e_T}{e_w} \left(\frac{b_T}{b} \right)^2 \right] \tag{8}$$

From Eq.(8) it can be seen that the maximum value of (e_0/e_w) is a strong function of (b_T/b) and it can take values greater than 1. Its value is 2 for $b_T = b$ and $e_T = e_w$, i.e. if the tail and wing have the same spans and similar spanwise load distributions.

A genralised expression for e_0 for n lifting surfaces placed together is derived in Ref.6 and accordingly e_0 for such a configuration is given by

$$e_0 = \sum_{i=1}^{n} \left(\frac{b_i}{b_R} \right)^2 e_i \tag{9}$$

where b_i's are the spans of individual surfaces and b_R is the span of the reference surface; e_i's are the individual pan form efficiency factors. Equation (9) shows that if n identical lifting surfaces are placed together the resulting value of e_o will be n times the individual value of e.

Conclusions

An airframe configuration can be considered as an assembly of several lifting surfaces each contributing to lift and induced drag. The nonlifting components do not contribute directly to either lift or induced drag and hence will have no direct effect on the value of e_o. Effects of components like fuselage which contribute to the lift and hence to the induced drag to a smaller extent are difficult to assess because for these components aspect ratios cannot be defined. The effects of these components would generally be that of reducing the value of e_o to the extent that their presence forces the spanwise load distributions on the main lifting surfaces to be more nonelliptic. Not withstanding these effects, it is seen that e_0 is a strong function of the horizontal tail span and if it is large enough we can still have a value of $e_o > 1$ for the configuration. It may therefore be concluded that having a value of e_o greater than unity is nothing strange, though one may not come across such configurations all the time.

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